Stable length on free factor, free splitting complexes and handlebody groups

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- Sphere graph: M^d manifold, vertices: $d-1$ dimensional embedded spheres, an edge of length 1 iff the spheres are disjoint.
- When $d=2$, this is the curve complex.
- Homeomorphism induces isometry on sphere graph
- Stable length: $l(f) := \liminf \frac{d(f^n(y),y)}{n}$.
- Known facts: rational (Bowditch), minimal $\sim g^{-2}$ (Gadre-Tsai), $\sim g^{-1}$ when restricted to Torelli (Baik-Shin).

- Fibered cone: Let f be a homeomorphism, N be the mapping torus, $\alpha \in H^1(N)$, there is a cone by Fried containing α where primitive integer points β corresponds to other fiberings over the circle, with fiber M_β and monodromy f_β.
- This is compatible with the fibered cone for surface maps by Thurston, and the symmetrized McMullen cone (or cone of section) by Dowdall-Kapovich-Leininger for doubled handlebodies.
- Theorem (Baik-Kim-W): Let L be a d-dimensional rational slice of any proper subcone of the fibered cone C (passing through 0), then for primitive integer points β in L, the sphere complex translation length $l(f_\beta) \lesssim \|\beta\|^{-1-1/(d-1)}$.

Applications

- Dimension 2: previously known by Baik-Shin-W.
- Dimension 3: for the case of doubled handlebody, this gives an upper bound on the free factor and free splitting complex stable translation length.
  - Free splitting complex: a simplex is an action of F_n on non trivial tree T with no F-inv subtrees, all edge stablizers trivial. Boundary map by collapsing edges.
  - Free factor complex: a simplex is a nested sequence of free factors. Boundary map by removing terms in the sequence.
- Same argument works for disc complexes on handlebodies. Results in examples of handlebody group elements with small stable length on curve complexes.