Groups of type FP_2: how many of them are there?

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A group G is of type …

$$1 \longrightarrow N \longrightarrow F \longrightarrow G \longrightarrow 1$$

F_2: **finitely presented:**
if N is finitely generated as a normal subgroup
(homotopical notion)

FP_2: **almost fin. presented:**
if N/N' is finitely generated as a $\mathbb{Z}G$-module
(homological notion)

Fact: $F_2 \subset FP_2$

Bestvina–Brady '97: $F_2 \neq FP_2$

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Q: How many groups of types F_2, FP_2 are there?

F_2:

$\aleph_0$ (countably many)

FP_2:

$2^{\aleph_0}$ up to isomorphism (Leary '18)
$2^{\aleph_0}$ up to quasi-isometry
(R.Kropholler–Leary–S. '20)

F_2:

(usual) **Dehn function** δ_G:

δ_G computable $\Rightarrow$ word problem is solvable for G

FP_2:

homological Dehn function FA_G:

$\exists$ groups with $FA_G(n) = n^4$ and unsolvable word problem (Brady–R.Kropholler–S. ’20)

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Theorem (N.Brady–R.Kropholler–S. '20)

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Open question: Characterize the homological isoperimetric spectrum

$$HIP = \{\alpha \mid n^\alpha \text{ is homological Dehn function of a group of type } FP_2\}.$$

Do we have equality $HIP = \{1\} \cup [2, \infty)$?