A version of omnipotence for virtually special cubulated groups

Sam Shepherd

Commanding group elements

Definition

A group G **commands** a set of elements $\{g_1, ..., g_n\} \subset G$ if there exists an integer $N > 0$ such that for any integers $r_1, ..., r_n > 0$ there exists a homomorphism to a finite group $G \to \bar{G}, g \mapsto \bar{g}$ such that the order of $\bar{g}_i$ is Nr_i.

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A group is **omnipotent** if it commands any independent set of elements $\{g_1, ..., g_n\}$ (i.e. the g_i have infinite order and no non-zero power of g_i is conjugate to a non-zero power of g_j for $i \neq j$).

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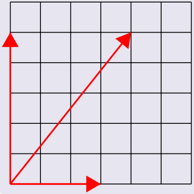
Examples of omnipotent groups:

- *free groups (Wise '00)*
- *hyperbolic surface groups (Bajpai '07)*
- *virtually special hyperbolic groups (Wise '12)*

My Theorem

A non-omnipotent group: $\mathbb{Z}^2$



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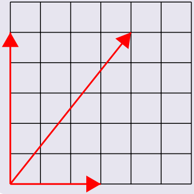

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An element g of a cubulated group $G \curvearrowright X$ is **convex** if it stabilises a convex subcomplex $Y \subset X$ with $Y/\langle g \rangle$ compact.

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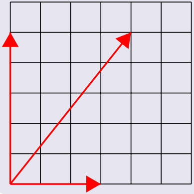

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Theorem

Every virtually special cubulated group $G \curvearrowright X$ commands every independent set of convex elements.