Kähler groups and finitely generated groups acting on trees

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We will be interested in the case when G is a surface group.

Dual tree of a simple closed curve in a closed oriented surface

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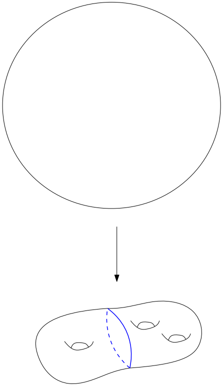

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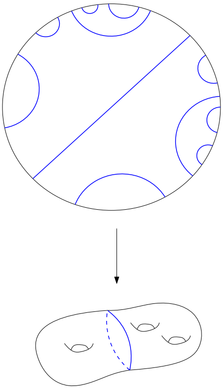

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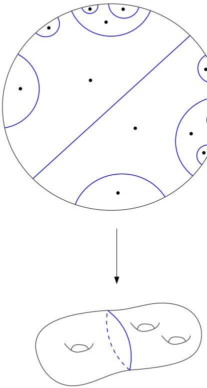

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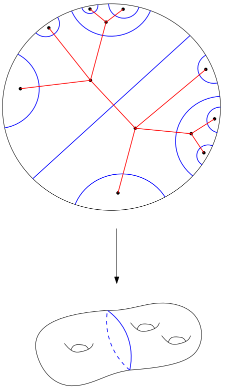

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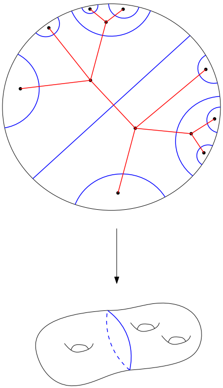

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The action $\pi_1(S) \curvearrowright \mathbb{H}^2$ induces an action $\pi_1(S) \curvearrowright T = (\mathcal{V}, \mathcal{E})$.

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Then there is a finite index subgroup Γ_1 of Γ containing $\pi_1(S)$ such that the restricted short exact sequence

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