Linear isoperimetric functions for surfaces in hyperbolic groups

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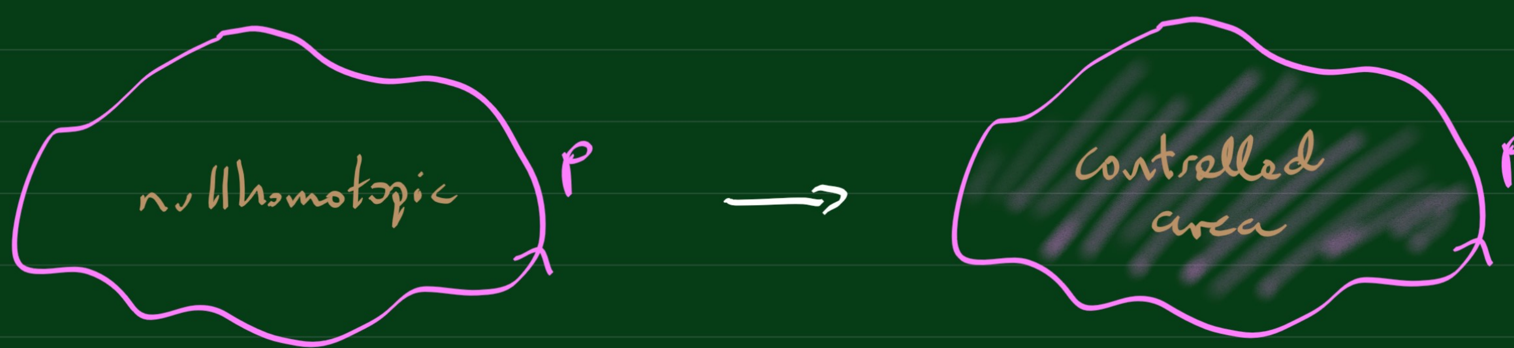
Young Geometric Group Theory X

Let G be a finitely presented group and X a 2-complex with $\pi_1 X \cong G$.

A map $f : \mathbb{N} \to \mathbb{N}$ is an **isoperimetric function** for X if for any nullhomotopic $p \to X$, there is a disc diagram $D \to X$ with $\partial D = p$ and

$$\text{Area}(D) \le f(|p|).$$



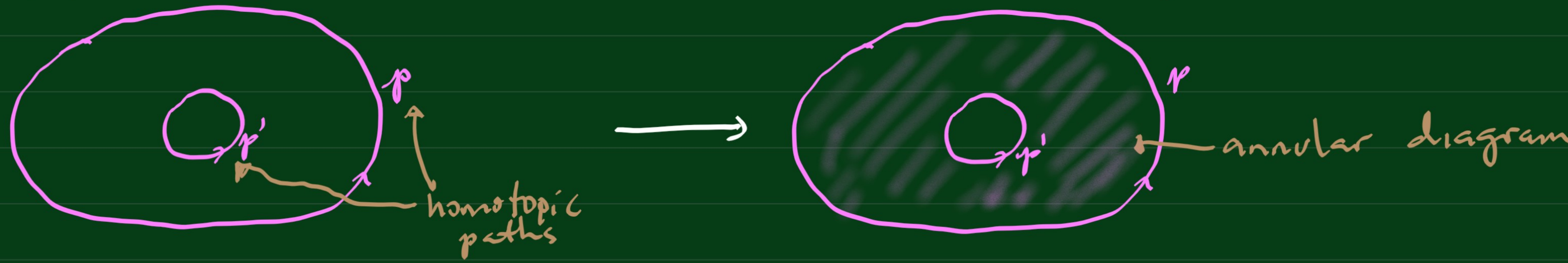
* Up to constants, minimal isoperimetric function $\text{Dehn}(G)$ is a group invariant, and in fact a Q.I. invariant.

Dehn functions quantify the word problem: Dehn(G) is recursive $\iff$ word problem is solvable.

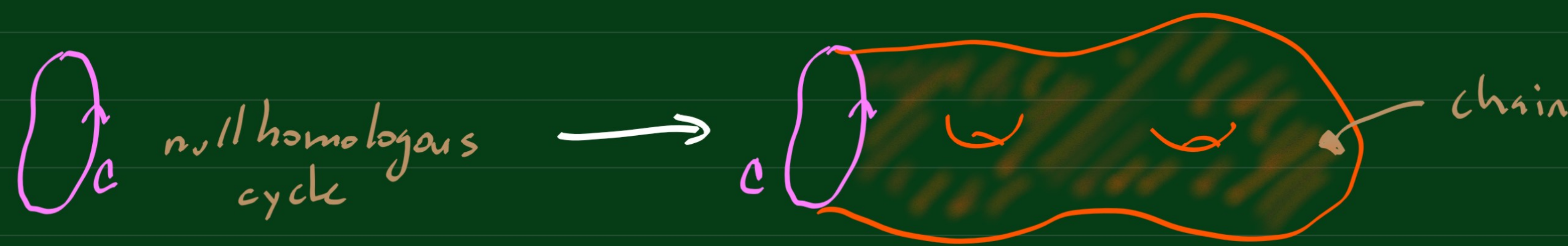
Theorem (Gromov) Dehn(G) is linear $\iff$ G is hyperbolic.

Examples:
- CAT(0) $\to$ Dehn is quadratic
- Nilpotent $\to$ Dehn is polynomial
- $BS(1,2)$ $\to$ Dehn is exponential

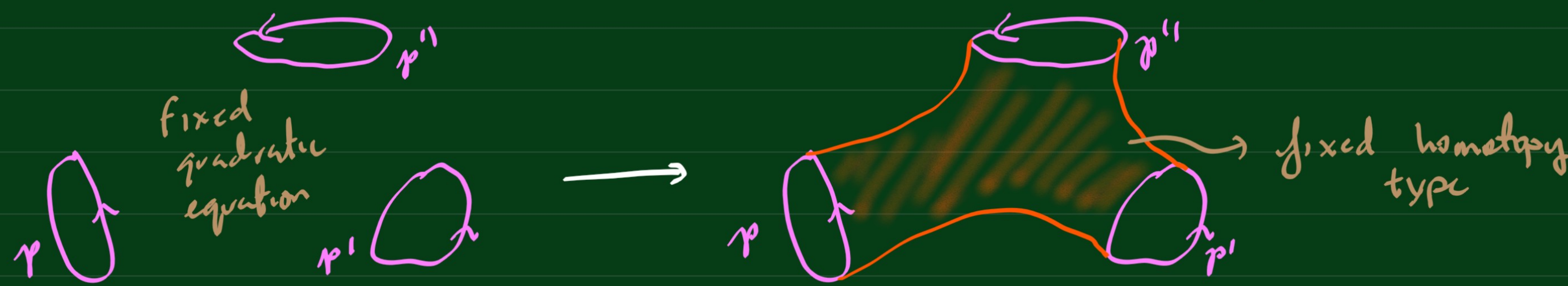
* Conjugacy problem $\longrightarrow$ annular isoperimetry



* "homological word problem" $\longrightarrow$ Homological isoperimetry



* quadratic equations $\longrightarrow$ Surface isoperimetry



Thm: G is hyperbolic $\iff$ Annular isp. function is linear.

Thm: G is hyperbolic $\Rightarrow$ homological isp functions are linear.

Thm: (A-Wise) G is hyperbolic $\Rightarrow$ "generalised" isp. functions are linear for <u>all</u> homotopy types of surface diagrams.