YOUNG GEOMETRIC GROUP THEORY X
NEWCASTLE UNIVERSITY

26-30 JULY 2021

TITLES AND ABSTRACTS OF INVITED TALKS

Mini Courses

Mladen Bestvina.

"Projection complexes and rotating families".

The celebrated work of Dahmani-Guirardel-Osin on rotating families proves a long standing conjecture that the normal closure of a suitable power of a pseudo-Anosov mapping class is free. Clay-Mangahas-Margalit, in a recent work, gave a version of rotating families for projection complexes (called spinning families), reproved the above application and gave several new applications. In the four lectures I will review the construction of projection complexes, and give proofs of [CMM] and [DGO] and applications. These proofs will follow "Free products from spinning and rotating families" by Bestvina, Dickmann, Domat, Kwak, Patel, and Stark.

Damian Osajda.

" Helly graphs and groups".

Abstract: A graph is Helly if any family of pairwise intersecting (combinatorial) balls has a nonempty intersection. This is a classical notion explored thoroughly within the metric graph theory, combinatorial optimization and elsewhere already for decades. Helly graphs are also known as absolute retracts - in the category of graphs with nonexpansive mappings. Recently, it has been shown that many classical groups act geometrically on Helly graphs - we call such groups Helly - leading to better understanding of those groups. This is due to the fact that Helly graphs, and hence Helly groups, exhibit many features of - widely understood - nonpositively curved objects. During the course we will present basic properties of Helly graphs and related Helly complexes, including their local-to-global characterization, which can be seen as an analogue of the classical Cartan-Hadamard theorem.

We will present relations between Helly graphs and injective metric spaces. The latter are, in many aspects, similar to CAT(0) spaces. We will also discuss the most important features of Helly groups, including biautomaticity, and numerous CAT(0)-like properties. On the way, we will present many examples of Helly groups and related objects. Finally, we will formulate important open questions and conjectures concerning the subject.

Rachel Skipper.

" Group Actions on Trees and Cantor Sets".

This series of talks will provide a tour of several families of exotic groups all arising via actions on trees and boundaries of trees (Cantor sets). We will start with the Grigorchuk group and other related subgroups of the automorphism group of the tree, including branch groups and automata groups. Then we will move to the family of Thompson groups and their various generalizations. Finally, we'll see how these fit together to form Neretin's group of spheromorphisms.

Todor Tsankov.

" Topological dynamics of countable groups".

Topological dynamics is the study of actions of infinite groups on topological (usually compact) spaces. The best understood case is that of actions of the group of integers, with many applications throughout mathematics. In this series of lectures, I will consider actions of general discrete groups and make an introduction to the more abstract facets of the theory. Topics covered will include:

- The enveloping semigroup and spaces of ultrafilters;
- Proximal flows and strong amenability;
- Non-free actions and uniformly recurrent subgroups.

Plenary Talks

Laura Ciobanu.

"Solving equations in groups".

For a group or semigroup or ring G, solving equations where the coefficients are elements in G and the solutions take values in G can be seen as akin to solving systems of linear equations in linear algebra, Diophantine equations in number theory, or more generally polynomial systems in algebraic geometry.

I will give a short survey about solving equations in infinite non-abelian (semi)groups, with emphasis on the free and hyperbolic ones. In particular, I will explain that the solutions to equations can be beautifully described in terms of formal languages, and that this was made possible by results coming from both geometry and computer science.

Maria Cumplido Cabello.

"Systolicity for large-type Artin groups".

Abstract: Artin groups are a natural generalisation of braid groups from an algebraic point of view: in the same way that braids are obtained from the presentation of the symmetric group, other Coxeter groups give rise to more general Artin groups. There are very few results proven for every Artin group. To study them, specialists have focused on some special kind of subgroup, called "parabolic subgroups". These groups are used to build important simplicial complexes, as the Deligne complex or the recent complex of irreducible parabolic subgroups. The question "Is the intersection of parabolic subgroups a parabolic subgroup?" is a very basic question whose answer is only known for spherical Artin groups and RAAGs. In this talk, we will see how we can answer this question in Artin groups of large type, by using the geometric realisation of the poset of parabolic subgroups, that we have named "Artin complex". In particular, we will show that this complex in the large case has a property called sistolicity (a sort of weak CAT(0) property) that allows us to apply techniques from geometric group theory. This is a joint work with Alexandre Martin and Nicolas Vaskou.

Dawid Kielak.

"The Friedl-Tillmann polytope".

First, I will give a brief introduction to one-relator groups. They form a very interesting family (studied since the very beginning of combinatorial group theory) that went through a somewhat dormant phase, but is back in focus now. There are a couple of new tools for studying these groups, and one of them is the Friedl-Tillmann polytope. I will define the polytope, and discuss the (solved) problem of the reliance of the definition on a particular presentation for the group.

The second topic I will discuss is how one can generalise the construction of the polytope to the L^2 polytope, and how this latter object is related to the Thurston polytope. Finally we will look into he role these three types of polytopes play in the theory of topological and algebraic fibring of 3-manifolds and free-by-cyclic groups.

Yash Lodha.

"Some new constructions and directions in the theory of left orderable groups".

Abstract: I will define two new constructions of finitely generated simple left orderable groups (in recent joint work with Hyde and Rivas). Among these examples are the first examples of finitely generated simple left orderable groups that admit a minimal action by homeomorphisms on the Torus, and the first family that admits such an action on the circle. I shall also present examples of finitely generated simple left orderable groups that are uniformly simple (these were constructed by me with Hyde in 2019). And present new examples that, somewhat surprisingly, have infinite commutator width.

Emily Stark.

"Commensurability and rigidity".

Abstract: Rigidity theorems prove that a group's geometry determines its algebra, typically up to abstract commensurability. A group's geometry is studied via its quasi-isometry class and via its geometric actions on proper geodesic metric spaces. These two points of view lead to two distinct notions of rigidity: quasi-isometric rigidity and action rigidity, respectively.

We will discuss the differences between these two notions of rigidity, focusing on groups constructed from manifold groups. Applications

include new examples of quasi-isometric groups that do not act geometrically on the same proper geodesic metric space. This is joint work with Alex Margolis, Sam Shepherd, and Daniel Woodhouse.

Robin Tucker-Drob.

"Measure Equivalence, Superrigidity, and Weak Pinsker Entropy".

Abstract: Measure Equivalence is an equivalence relation on countable groups introduced by Gromov as a measure-theoretic counterpart to the relation of quasi-isometry coming from geometric group theory. In the first part of this talk I will give an introduction to measure equivalence, focusing on examples and open problems. In the second part of the talk I will explain Popa's Cocycle Superrigidity Theorem and discuss work with Lewis Bowen in which we show that the collection of groups which satisfy the conclusion of Popa's Theorem is invariant under measure equivalence. Time permitted, I will also indicate some consequences of our result to a new notion of entropy called weak Pinsker entropy.

Tianyi Zheng.

"Random walks on lamplighters and related groups".

Abstract: Random walks on lamplighter groups were first considered by Kaimanovich and Vershik to provide examples of amenable groups with nontrivial Poisson boundary. Such processes can be understood rather explicitly, and provide guidance in the study of random walks on more complicated groups. We will discuss a law of large numbers for random walks on the two-dimensional lamplighter group, and various questions regarding limiting behavior of random walks and the geometry of groups. Joint work with Anna Erschler.