Newcastle University

Book of Abstracts

The 65th British Applied Mathematics Colloquium

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Welcome

We are delighted to welcome you to the 65th British Applied Mathematics Colloquium (BAMC 2024); the largest applied mathematics conference in the UK bringing together students, academics, and industry professionals to discuss recent advances within the field.

All abstracts within this booklet (except for plenaries) are numbered corresponding to their number on the programme. They are further categorised into contributed talks, mini-symposia, and posters. We hope you enjoy the fantastic programme on offer and thank everyone who has submitted an abstract or lunchtime workshop.

In partnership with Beacon House Events we are committed to minimising the impact of this event on the environment. We have worked closely with our venues, suppliers, and delegates to reduce the impact of our activity wherever possible. We are doing this by limiting printed materials, opting for low impact catering options and tableware among other things.

Finally we'd like to thank all our sponsors for their generous support of the BAMC 2024 and everyone who is participating in the event both online and in person.

We wish you a very warm welcome to Newcastle,

Andrew Baggaley, Paul Bushby, Magda Carr (Chair), Matthew Crowe, Otti Croze, Ryan Doran, Celine Guervilly, Graeme Sarson, David Swailes, Yue-Kin Tsang, Cora Uhlemann, Laura Wadkin & Toby Wood.

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Abstracts of Plenary Speakers

The mathematics and physics of wound healing

Professor Tanniemola Liverpool

University of Bristol

Abstract:

I will discuss our work looking quantitatively at the process of wound healing using ideas from thermodynamics and statistical mechanics. Wound healing is a highly conserved process required for survival of an animal after tissue damage. The wound repair process is not only of great interest in its own right but is also a laboratory to study complex tissue dynamics and regeneration.

Many wounds involve damage to an epithelial (barrier) tissue (like skin) that separates different regions of the body of a living organism. I will describe some recent work on studying wound healing in two dimensional epithelial tissues of a fruit fly pupal wing. This epithelium was chosen because it is transparent and accessible to sophisticated imaging techniques. We use live confocal time-lapse microscopy to follow the behaviour of cells in a tissue before and after wounding.

I will focus on three cell-behaviours that are generally accepted to contribute to wound re-epithelialisation: cell shape deformation, cell division, and cell migration.

I will describe how we are beginning to use a combination of mathematics, physics and biology to disentangle some of the organising principles behind the complex orchestrated dynamics that lead to wound healing.

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PLENARY ABSTRACT

Controlling flow through viscosity manipulation

Professor Nigel Mottram

University of Glasgow

Abstract:

The flow of a fluid around a solid object is an everyday phenomenon – water flowing around the rocks in a stream, air being funneled through a gap between buildings – and changing the shape of a solid object allows active manipulation of the direction of flow – ailerons and the rudder on an aircraft work in exactly this way. But in a microfluidic device, where we consider extremely small volumes of liquid, moving at sub-millimeter lengthscales, moving solid objects to affect the flow can be difficult, with fine control of the flow being challenging.

However, the active control and manipulation of flow streamlines, as well as interfaces between fluids, at micron and millimeter scales often plays an important role in many industrial and technological applications, and so significant effort has gone into this area of research. In this talk I will present some recent work on how liquid crystal materials offer a new way to tackle these challenges. I will begin with an introduction to the world of liquid crystals – which are viscoelastic liquids in which microstructure and flow are intrinsically linked – and then show how, through electrostatic manipulation of the microstructure and thus the viscosity, it is possible to create controlled manipulation of streamlines and free-surfaces – equivalent to placing a solid or porous object in the way of the flow, with the added benefit of being able to actively alter the porosity. In this work we have considered two examples to demonstrate the effect: flow of a liquid crystal rivulet down an inclined substrate, in which this viscosity manipulation effects lead to changes in the rivulet height, equivalent to the rivulet flowing over an obstacle; and Hele-Shaw flow, in which viscosity manipulation leads to tailoring of streamlines around a virtual porous object. In both cases we have confirmed our theoretical results using equivalent experiments. This effect, and the fine control possible with liquid crystals, has the potential for active control of many more fluid systems such as viscous fingering, particle transport, mixing and coating flows.

This work is a collaborative project involving a team of theoreticians at Glasgow and Strathclyde (Dr J.R.L. Cousins, Prof. S.K. Wilson and Dr B.R. Duffy) and a team of experimentalists at Nottingham Trent (led by Dr A.S. Bhadwal and Prof. C.V. Brown) and has been supported by the EPSRC, through research grants EP/P51066X/1 and EP/T012501/2, and Merck KGaA

Regularity of Turbulence: numerical and experimental explorations

Dr Bérengère Dubrulle

CNRS, SPEC, University Paris-Saclay, France

Abstract:

Turbulence, a phenomenon observed by physicists in natural and laboratory flows, is thought to be described by Navier-Stokes equations (NSE). Mathematicians are wondering whether NSE are well posed, namely whether they can develop a singularity in finite time from regular initial conditions. Due to proliferating degrees of freedom, direct numerical simulations of the NSE are not ideal to investigate such issues.

In this talk, I will discuss new numerical and experimental tools that can be used to explore regularity of turbulence.

*From a numerical point of view, I will show how projection of the equations of motions on log-lattices enables to detect and characterize finite-time blows-up for hyper, hypo and viscous Navier-Stokes, and their potential link with weak dissipative solutions of Euler equations.

*From an experimental point of view, I will show how progresses in image velocimetry has made it possible to investigate the nature and the properties of small-scale turbulent motions, at scales of the order of or below the Kolmogorov scale. In particular, I will show how the notion of "inertial dissipation" can be used to detect areas of lesser regularities of the velocity field, and discuss their connection with vortex reconnection and irreversibility.

References:

Cheminet et al, PRL, 129, 124501 (2022)

Dubrulle, B. J. Fluid Mech, 867:1, (2019)

Pikeroen et al, submitted to nonlinearity (2023)

The 65th British Applied Mathematics Colloquium

PLENARY ABSTRACT

From Maths to Policy: a Covid 19 Story

Dr Julia Gog

Cambridge University

Abstract:

Unfortunately, modelling of infectious disease no longer needs as much introduction as it did before 2020. The use of mathematical approaches during the COVID-19 pandemic was very visible, both for its role in scientific advice for policy decisions, and for the public understanding of the unfolding pandemic.

This talk will give a speed background to the roles of mathematical modelling, and some of the insights that epidemic models offer. Putting this together with COVID-19 we'll look at how modelling was able to help contribute to scientific advice to the UK government, and something of one mathematician's experiences during these times. The pandemic also highlighted the importance of science communication to the public, as well as leaving us with some research challenges for the future.

PLENARY ABSTRACT

Exploring Quantum Liquids as Simulators for Black Hole Processes

Dr Silke Weinfurtner

University of Nottingham

Abstract:

In this presentation, I will delve into the fascinating analogy between the dynamics of small perturbations in quantum fluids and quantum field theory in curved spacetimes. Through this discussion, I will outline the essential requirements for utilizing quantum fluids as simulators for black hole processes. To illustrate these concepts, I will present recent findings on simulating black hole processes in superfluid helium 4, where a giant quantum vortex flow manifests. This exploration promises to shed light on the potential of quantum liquids as powerful tools for studying and understanding complex phenomena in black hole physics.

PLENARY ABSTRACT

Rapidly Rotating Magnetohydrodynamic Flows and the Geodynamo

Dr Emmanuel Dormy

Ecole Normale Supérieure

Abstract:

The problem of the Geodynamo is apparently simple to formulate ("Why does the Earth possess a magnetic field?") yet it proves surprisingly hard to address. As most geophysical flows, the fluid flow of molten iron in the Earth core is strongly influenced by the Coriolis force. Because the liquid is electrically conducting, it is also strongly influenced by the effects of the Lorentz force. The balance is unusual in that, whereas each of these effects considered separately tends to impede the flow, the magnetic field in the Earth core relaxes the effect of the rapid rotation and allows the development of a large-scale flow in the core which in turn regenerates the field. This presentation intends to review recent developments and current understanding on the interplay between rotation and magnetic field.

Abstracts of Contributed Talks

How to measure the controllability of an infectious disease?

Dr Kris Parag

Imperial College London

Abstract:

Background Reproduction numbers, R, and growth rates, r, are frequently used to assess how difficult it is to control the spread of an infectious disease. An uncontrolled outbreak with larger R or r is widely interpreted as less controllable because, supposedly, we must block (R-1)/R infections or halve infections within (1/r)log(2) time units to stabilise transmission.

Results
We demonstrate that these interpretations are unrealistic, being only valid under the idealistic conditions that interventions are enacted without delays and change transmission parameters without inducing additional dynamics. While the signs of R-1 or r delineate epidemic stability, their magnitudes are unreliable and incomplete measures of the distance from stability i.e., outbreaks with larger values are not always intrinsically less controllable and those with smaller values are not necessarily more robust. Using classical control theoretic margins, which explicitly consider feedback properties of epidemics under interventions, we propose a comprehensive framework for measuring controllability. This precisely describes how much we can scale infections or delay interventions before stability is lost.

Implications
Applying our framework, we find that pathogens with similar R or r can have markedly different controllability and find decision boundaries for transitioning from targeted (e.g., quarantines) to wide-scale interventions (e.g., lockdown). Our framework unifies, within two parameters, how important factors such as pre-symptomatic spread, super-spreading and surveillance imperfections all impact infectious disease controllability. While R and r remain important for tracking transmission, we assert that reliable measures of controllability must reflect the complex feedback loops between interventions and their implementation.

The effects of wall compliance on the stability of jets and wakes

Mr Ryan Poole

University of Surrey

Abstract:

In this talk, we examine the local stability properties of two-dimensional planar jets and wakes which are confined by two identical 'Kramer-type' compliant boundaries. The basic flow profile is modelled as a three-part piecewise linear flow profile, while the compliant walls are modelled as a spring backed elastic plate. We derive a dispersion relation $\Delta(\omega,\alpha)=0$ for both varicose and sinuous modes of the system. Here ω is the angular frequency of the stability waves, where in particular $\mathrm{Im}(\omega)$ describes the wave growth rate, and α is the wavenumber.

We perform a spatio-temporal stability analysis on the system, for a range of flow and wall configurations. It was found that compliant walls are shown to modify existing shear-induced instabilities, as well as introduce new additional instabilities originating from the presence of the wall itself. These new modes can dominate the flow stability characteristics and can lead to unstable behaviour at lower shear values than for rigid walls.

To confirm that these new wall-induced modes are not a consequence of the piecewise linear base flow, we examine a smoothed version of this base flow for comparison. We find that the results of the smooth base flow calculations are in qualitative agreement with those of the piecewise linear base flow, hence the observed behaviour is expected in real flows.

The final part of the talk looks at how the stability properties of this flow are affected by breaking the flow symmetry by non-identical walls or asymmetric confinement.

Modeling and transmission dynamics of Zika virus through efficient numerical method

Dr Ali Raza

Higher Education Department, Govt of Punjab, Pakistan

Abstract:

Zika virus infection is a vastly transmitted disease among humans. It was carried worldwide by international travelers. In 2016, Zika virus infection was present in more than 20 countries and territories in America. Thousands of cases were diagnosed in Cabo Verde, western Africa.
Fifty-seven regions suffered from Zika virus in 2020, and the World Health Organization reported more than one hundred thousand cases worldwide. In this work, the modeling and transmission dynamics of Zika virus are studied dynamically and numerically. Positivity, boundedness, reproduction number, equilibria, and local stability are part of the numerical analysis. New nonstandard numerical techniques are examined for the said model. The primary purpose is to maintain the continuous model's behavior and dynamical properties. The proposed nonstandard finite approximation is studied according to the consistency and local stability of the solutions. Some numerical examples clearly show the improvement of the new schemes compared to other well-known methods"

Intelligent Optimization Analysis of the Cholera Epidemic Model

Dr Tahir Nawaz Cheema

Pakistan Model Higher Secondary School for Girls Saroke Cheema, Wazirabad

Abstract:

Cholera is a global threat to public health and is an indicator of inequity and lack of social development. By the World Health Organization (WHO), there are 1.3 to 4.0 million cases of Cholera and 21,000 to 143,000 deaths worldwide due to the infection each year. This innovative work discusses the spread of the Cholera virus; the model of this disease was formulated mathematically and solved with the help of the Artificial Neural Networks technique. The developed model identified the nonlinear ordinary differential equations represented by susceptible (Sc), vaccinated (Vc), infectious (Ic), recovery (Rc), and concentration of Cholera in water (Bc), and the Cholera model reference dataset is formed using the explicit Runge-Kutta method.The Levenberg-Marquardt back-propagation is implemented to refine the data set of the Cholera model for training, testing, and validation. The accuracy of the proposed technique is evaluated through mean squared error (MSE), error histograms, merit functions, reliable performance and regression.

Square-Triangle periodic approximants in block copolymer phase separation.

Dr Merin Joseph

University of Leeds

Abstract:

When one or more geometrical entities can fill a 2D space without any gaps or overlapping, we call that tiling. These tilings can be spatially periodic or aperiodic. Aperiodic tilings are associated with quasicrystals - materials with long-range order but without the usual spatial periodicity of crystals. Quasicrystals are found in soft matter systems like colloids, liquid crystals, block copolymers, etc. Block copolymer melts show the phenomenon of phase separation into different morphological patterns depending on their molecular architecture and chemical incompatibility. Phase separation gives rise to a wide variety of periodic and aperiodic structures. In 2D, phase separation is analogous to tilings. Moreover, dodecagonal quasicrystalline tilings were observed by Hayashida et al. in ABC star block copolymers. Polymer phase separation theories like strong segregation theory (SST) can be used to study the stability of morphologies formed in these melts. In our work, we explain how we use SST to analyze the stability of different morphologies found in phase-separated ABC star terpolymers. The aim is to develop a geometrical framework using SST that can incorporate both periodic tilings and periodic approximants of aperiodic tilings and develop the phase space for ABC star terpolymers, potentially generating 12-fold square triangle tilings.

[1] Kenichi Hayashida, Tomonari Dotera, Atsushi Takano, and Yushu Matsushita. Polymeric Quasicrystal: Mesoscopic Quasicrystalline Tiling in A B C Star Polymers. Physical Review Letters, 98(19)

[2] Tohru Gemma, Akira Hatano, and Tomonari Dotera. Monte Carlo simulations of the morphology of ABC star polymers using the diagonal bond method. Macromolecules, 35(8):32253237, 2002.

Quantifying Cytoskeletal Dynamics and Remodeling from Live-imaging Microscopy Data

Miss Carey Li

University of St Andrews

Abstract:

The shape of biological cells emerges from dynamic remodeling of the cell's internal scaffolding, the cytoskeleton. Hence, correct cytoskeletal regulation is crucial for the control of cell behaviour, such as cell division and migration. A main component of the cytoskeleton is actin. Interlinked actin filaments span the body of the cell and contribute to a cell's stiffness. The molecular motor myosin can induce constriction of the cell by moving actin filaments against each other. Capturing and quantifying these interactions between myosin and actin in living cells is an ongoing challenge. For example, live-imaging microscopy can be used to study the dynamic changes of actin and myosin density in deforming cells. These imaging data can be quantified using Optical Flow algorithms, which locally assign velocities of cytoskeletal movement to the data. Extended Optical Flow algorithms also quantify actin polymerization and depolymerization. However, these measurements on cytoskeletal dynamics may be influenced by noise in the image acquisition, by ad-hoc parameter choices in the algorithm, and by image pre-processing steps. The development of our Optical Flow method will be a starting point for identifying differences in cytoskeletal movement and remodeling under experimental perturbations.

On a Dynamic Variant of the Regularized Gauss-Newton Method with Sequential Data

Dr Neil Chada

Heriot Watt University

Abstract:

For numerous parameter and state estimation problems, assimilating new data as they become available can help produce accurate and fast inference of unknown quantities. While most existing algorithms for solving those kind of ill-posed inverse problems can only be used with a single instance of the observed data, in this work we propose a new framework that enables existing algorithms to invert multiple instances of data in a sequential fashion. Specifically we will work with the well-known iteratively regularized Gauss-Newton method (IRGNM), a variational methodology for solving nonlinear inverse problems. We develop a theory of convergence analysis for a proposed dynamic IRGNM algorithm in the presence of Gaussian white noise. We combine this algorithm with the classical IRGNM to deliver a practical (hybrid) algorithm that can invert data sequentially while producing fast estimates. Our work includes the proof of well-definedness of the proposed iterative scheme, as well as various error bounds that rely on standard assumptions for nonlinear inverse problems. We use several numerical experiments to verify our theoretical findings, and to highlight the benefits of incorporating sequential data. The context of the numerical experiments comprises various parameter identification problems including a Darcy flow elliptic PDE example, and that of electrical impedance tomography.

A Continuum Level model for Sintering

Dr Mat Hunt

Qdot Technology

Abstract:

Modern engineering requires increasingly intricate geometries for optimum performance. One way to obtain such intricate geometries is through a process known as co-sintering. Sintering is a heat treatment process that involves subjecting aggregate material to temperature and pressure to compact the loose material into a solid object. Co-sintering is the process of sintering two materials together as a component. In this talk, we shall set out modelling co-sintering via continuum mechanics, deriving a mathematical model describing the dynamic change in temperature, density and velocity. The main variable of interest is the porosity defined as $\rho=\rho_{0}(1-\theta)$which is a measure of the gaps within the material. The change in geometry due to the sintering process is also of great interest. The model we analyse numerically is isothermal and we compute the 1D and 2D solutions and see how much the geometry changes through co-sintering.

Wave-analysis in a rotating transversely isotropic thermoelastic diffusion solid half-space in a higher-order fractional and memory dependent elasticity.

Dr Anand Kumar Yadav

Shushi Niketan M. S. S. S. Sector 22D, Chandigarh, India

Abstract:

For numerous parameter and state estimation problems, assimilating new data as they become available can help produce accurate and fast inference of unknown quantities. While most existing algorithms for solving those kind of ill-posed inverse problems can only be used with a single instance of the observed data, in this work we propose a new framework that enables existing algorithms to invert multiple instances of data in a sequential fashion. Specifically we will work with the well-known iteratively regularized Gauss-Newton method (IRGNM), a variational methodology for solving nonlinear inverse problems. We develop a theory of convergence analysis for a proposed dynamic IRGNM algorithm in the presence of Gaussian white noise. We combine this algorithm with the classical IRGNM to deliver a practical (hybrid) algorithm that can invert data sequentially while producing fast estimates. Our work includes the proof of well-definedness of the proposed iterative scheme, as well as various error bounds that rely on standard assumptions for nonlinear inverse problems. We use several numerical experiments to verify our theoretical findings, and to highlight the benefits of incorporating sequential data. The context of the numerical experiments comprises various parameter identification problems including a Darcy flow elliptic PDE example, and that of electrical impedance tomography.

Cost optimisation of hybrid institutional incentives for promoting cooperation in finite populations

Miss Calina Durbac

University of Birmingham

Abstract:

We rigorously study the problem of cost optimisation of hybrid (mixed) institutional incentives, which are a plan of actions involving the use of reward and punishment by an external decision-maker, for maximising the level (or guaranteeing at least a certain level) of cooperative behaviour in a well-mixed, finite population of self-regarding individuals who interact via cooperation dilemmas (Donation Game or Public Goods Game). We show that a mixed incentive scheme can offer a more cost-efficient approach for providing incentives while ensuring the same level or standard of cooperation in the long-run. We establish the asymptotic behaviour (namely neutral drift, strong selection, and infinite-population limits). We prove the existence of a phase transition, obtaining the critical threshold of the strength of selection at which the monotonicity of the cost function changes and providing an algorithm for finding the optimal value of the individual incentive cost. Our analytical results are illustrated with numerical investigations.

Predicting low-speed rarefied gas flow in a lid-driven cavity using a supervised machine learning approach

Dr Arshad Kamal

University of Warwick

Abstract:

Multi-scale modelling that combines a continuum-based macro-model with a particle-based micro-model is often required to model low-speed rarefied gas flows in MEMS devices due to the wide range of length and time scales. The Direct Simulation Monte Carlo (DSMC) method, the micro-model, can be used in the non-equilibrium regions to improve accuracy, whereas the macro, Computational Fluid Dynamics (CFD) can be used in the remaining regions to improve computational efficiency. However, the DSMC method produces significant statistical noise in estimating macroscopic properties, which can lead to numerical instabilities in the coupling algorithm when transferring the noisy data into a CFD model. This can be avoided by using more particles per cell or running over a larger sampling window. Both approaches, however, increase the computational cost. To overcome this issue and reduce the reliance on the DSMC, we introduce noise-free surrogate models that are designed to find the boundary conditions and stress corrections required by the CFD model. We employ relevance vector machines, a sparse Bayesian formulation, to learn the slip velocities and stress corrections from noisy, target DSMC data. These corrections are then incorporated into the CFD, which acts as a substitute for the DSMC in the non-equilibrium regions. We apply this method to the 2D steady lid-driven cavity problem for a range of Mach and Knudsen numbers. We observe our method is computationally efficient in removing statistical noise and finding corrections from the target DSMC, producing results in good agreement with benchmark data.

Post-operative monitoring of human corneal cells based on in-vivo confocal microscopy study

Mr Patrick Parkinson

Newcastle University

Abstract:

Limbal stem cells (LSCs) play a vital role in corneal epithelial maintenance, cell renewal, and wound healing. Limbal stem cell deficiency (LSCD) poses a threat to vision, motivating innovative therapies such as ex-vivo cultured transplants. Our work aims to investigate the efficacy of LSC transplants for treating LSCD through computational and mathematical modelling.

We aim to determine the diagnostic significance of various cell characteristics in post-operative cornea monitoring, through thorough statistical analysis of IVCM images from 9 LSCD patients pre- and post-treatment.

We utilise ImageJ's morphological segmentation algorithm to extract cell characteristics such as cell area, number of neighbours, and aspect ratio. Upon analysis, we find that the distribution of cell areas emerges as a meaningful diagnostic parameter, demonstrating significant variation between healthy and damaged corneas. Furthermore, treated LSCD eyes exhibit a recovery in cell area distribution over 24 months, suggesting its utility as an informative metric for diagnosis and post-treatment monitoring. Contrastingly, the number of neighbours, indicative of cell shape, remains consistent across healthy, LSCD-affected, and recovering corneas. This challenges the diagnostic relevance of cell shape in LSCD and post-operative monitoring.

These investigations will assist in the development of computational diagnostic tools for the monitoring of LSCD, and advance agent-based computational models of corneal wound healing. This research enhances our understanding of LSC transplant outcomes and lays the foundation for improved diagnostics and monitoring strategies for LSCD patients.

Biofilm growth under localized antimicrobial treatment

Miss Parna Mandal

University of Glasgow

Abstract:

Bacteria can thrive within surface-bound communities called biofilms, evolving through stages such as initial attachment, stabilisation through extracellular polymeric substance (EPS) production, and development of different bacterial phenotypes such as proliferative, persister and dead cells, and finally possible detachment. Single-species biofilms are typically associated with medical implant infection and treating these infections is challenging due to the biofilm's resistance to antibiotics, compounded by issues like nutrient deficiency and low oxygen in deeper biofilm layers. Our study presents a mathematical model to understand how biofilms react to varying antibiotic doses and release rates at the implant-biofilm interface, aiming to refine targeted drug treatments.

Our one-dimensional model for biofilm growth allows for controlled antibiotic release from implant and includes different bacterial phenotypes, EPS, nutrient levels, biofilm water content, biofilm growth, and a porous implant filled with antibiotic. This model has already advanced our understanding of effective drug delivery, considering factors like nutrient availability and the development of bacterial phenotypes. The model also looks at the effects of different antibiotic release methods from polymer-free, nano-porous implants, considering both natural and antibiotic-induced bacterial mortality. Notably, as antibiotic doses increase, the density of proliferative bacteria decreases, and persister bacteria increases, similar to experimentally observed antibiotic resistance. Our next goal is to identify the optimum antibiotic administration method to eliminate both the infection and these resilient cells, preventing further implant infections.

Inferring the Utility from Optimal Behaviour in an Epidemic using Neural Networks

Mr Mark Lynch

University of Warwick

Abstract:

Physical, chemical, biological, and social systems can be understood as particular dynamical systems, in which the time-evolution of the state follows a given rule or law. Many of these systems can be represented as "differential games" where different interacting individuals are each seeking to simultaneously maximise their own utility function by modifying their behaviour. Here we consider rational individuals socially distancing in an epidemic. Given a specific form of utility, one can solve the related constrained optimal control problem to derive optimal system dynamics that result in the maximal utilities for each individual. We seek to use Machine Learning techniques to solve the inverse problem, that of inferring some unknown utility function that is being optimised by given system dynamics. Usually this has been solved by assuming some fixed form of the utility. We propose a more ambitious machine learning framework that is able to infer this hidden utility assuming no knowledge of the form of this function. The main issue to address is how to perform the learning of such a function using solely measurable data (the state of the system at any given time), that is, without knowledge of the hidden variables required to define the underlying constrained optimization problem (i.e., the Lagrange multipliers). Once the method has been established we will be able to analyze real-world epidemics data, observing the number of susceptible and infected individuals over time, and investigate the hidden utility functions that were being optimised for by the population.

Global stability of a confined shear layer with corrugated boundary

Dr Matt Turner

University of Surrey

Abstract:

This talk investigates the global instability properties of a shear layer confined above by a flat free surface and below by a spatially periodic, corrugated, impermeable boundary. Using linear stability theory, the instability properties are investigated for a piecewise-linear approximation. It is found that the global instability is driven by a resonance process from a boundary with a wavelength which also corresponds to a temporally unstable shear layer. For bottom boundaries with wavelengths outside this range we find the global instability can be found by investigating special saddle points in the complex wavenumber plane which arise due to the coupling of modes in the Floquet solution expansion.

General Models for Transcription Factor Binding to DNA Promoters with Double Binding Site

Miss Hanzhen Shen

University of Nottingham

Abstract:

We've explored the scenario where a single transcription factor attaches to a DNA promoter site. In more prevalent instances, DNA promoters may encompass multiple sites that can function autonomously or interact in unison, thereby multiple transcription factors can engage with distinct promoter sites. The coordination of genes is frequently facilitated by the collaboration of various transcription factors. We elucidate the combined influence of these regulators through a multidimensional input mechanism. In this paper, we presume that both gene X1 and gene X2 play regulatory roles in gene Y. Moreover, each DNA strand of gene Y features two distinct promoter sites. The regulatory action on gene Y is generated by the transcription factors of X1 and X2, which engage with the two DNA promoter sites of Y. We explore the different binding behaviors of transcription factors when they bind to designated promoter sites versus the scenario where they could potentially bind to any random promoter site, where these promoter sites are assumed to be independent.

Fast Bayesian Identification of Nonlinear Dynamics (BINDy) in scarce and noisy data

Dr Lloyd Fung

University of Cambridge

Abstract:

The Sparse Identification of Nonlinear Dynamics (SINDy) framework has been shown to be effective in learning interpretable and parsimonious models directly from data. However, existing SINDy derivatives can be computationally expensive and may struggle to learn the correct model equations from noisy and small datasets.

We propose a Bayesian extension to SINDy for learning sparse equations from data. Our method shows more robust capability in learning the correct model in the low-data limit, as it sparsifies the model based on both the value and the distribution of the parameters during regression, and uses the marginal likelihood (evidence) to rank and select the candidate models. The proposed method uses Laplace's method to approximate the Bayesian likelihood and evidence, avoiding the need for computationally expensive Markov chain Monte Carlo (MCMC) sampling. This results in a significant speedup in computation compared to other Bayesian SINDy methods, while still achieving comparable or better performance than existing methods such as Ensemble-SINDy.

We demonstrate the effectiveness of the proposed method on a variety of problems, including learning the Lotka-Volterra equations from 21 experimental data points of the Hudson Bay Lynx-Hare population dataset, and the Lorenz system using tens of noisy data points.

Shapes optimising grand resistance tensor entries in a Stokes flow

Dr Clément Moreau

CNRS

Abstract:

Stokes flow characterises the behaviour of a fluid for which viscosity effects predominate over inertial effects, which is typically the case at microscopic scale. At this regime, there is a linear relationship between the motion of a body and the hydrodynamic forces it experiences. This relationship is materialised by the grand resistance tensor, which contains a finite number of parameters depending intrinsically on the shape of the object, and notably not on the boundary conditions of the fluid flow.

Hence, it makes sense to optimise the shape of a body with respect to one or several of these parameters, to achieve various objectives related to hydrodynamic drag. Solutions were found for optimal translational drag and more recently in particular cases of elongated bodies, but the problem remains open in all generality.

In this talk, I will present a theoretical approach to obtain optimised shapes for this problem, using the framework of Hadamard shape derivatives, which has the advantage of working for a very general class of shapes. I will show that the optimisation problem amounts to computing a single, simple formula for the shape derivative, which depends on the solution of two Stokes problems with well-chosen boundary conditions. Then, I will address some issues about the numerical implementation of a suitable optimisation algorithm and discuss numerical results, including the existence of many local minima and the emergence of chirality.

This work was conducted in collaboration with Prof. Kenta Ishimoto (Kyoto University) and Prof. Yannick Privat (Université de Strasbourg).

Bridging the gap between agent-based models and continuous opinion dynamics

Mr Andrew Nugent

University of Warwick

Abstract:

There is a rich literature on microscopic models for opinion dynamics; most of them fall into one of two categories - agent-based models or differential equation models. Agent-based models more closely mirror real life interactions: randomly chosen agents meet in pairs and may or may not change their opinions at that specific time. By contrast, in differential equation models, individuals can interact constantly with the entire population and continually update their opinions. We describe how differential equation models can be obtained from agent-based models by simultaneously re-scaling time and the distance by which agents update their opinions. The pathway this approach provides not only gives a rigorous justification of differential equation models, but also a route to analyse modelling choices. For example: it motivates several possible multiplicative noise terms in stochastic differential equation models; clarifies the connection between selection noise and the mollification of the discontinuous bounded confidence interaction function; and shows how the method for selecting interacting pairs can determine the normalisation in the corresponding differential equation.

Drying of porous media with impurities

Dr Ellen Luckins

University of Nottingham

Abstract:

As a filter (eg: in a washing machine) or a textile (eg: a raincoat) dries, any dirt or impurities in the water are left behind within the filter or the textile. It's important to know where the dirt goes because it impacts the future efficacy of the filter or raincoat: does it deposit uniformly throughout the porous material, or concentrate at the surface? Can the accumulating dirt ever clog the porespace entirely, and if so, how can we avoid this? To investigate, we derive and solve a model for the motion of an evaporation front through a porous material, coupled with the accumulation, transport and deposition of the dirt.

Good things come in threes: the interface dynamics of drops impacting onto a different liquid

Dr Radu Cimpeanu

University of Warwick

Abstract:

In this talk we explore drop impact onto a pool of another liquid, a scenario of relevance to many applications of interest, from inkjet printing to estimating environmental risks after oil spills. A combination of high-speed photography, direct numerical simulation, and mathematical modelling has been used to disentangle the different roles that physical fluid properties play in determining the detailed dynamics in this three-phase multi-fluid system. Simple mechanistic models and experiments of drops impacting onto a pool of the same fluid have led to estimates of the penetration speed being half the impacting drop speed. However, this is only one small part in a rich and intricate behaviour landscape once fluid properties are no longer identical - we uncover velocities between 10% and 90% of the reference initial drop velocity over a range of three orders of magnitude in density and viscosity ratios between the impacting drop and the pool, while also explaining these findings and summarising them into one key compact predictive formula (Physical Review E 104 (6), 065102, 2021). In higher speed contexts we then shed light into changes to the splashing threshold within the more complex landscape of three-phase flows (Journal of Colloid and Interface Science 641, 585-594, 2023), before ultimately turning to an intermediate system consisting of a drop impacting a pool covered by a thin liquid film, as often found in practical applications of interest.

Information geometry of evolution of neural network parameters while training

Mr Abhiram Thiruthummal

Coventry University

Abstract:

Artificial neural networks (ANNs) are powerful tools capable of approximating any arbitrary mathematical function, but their interpretability remains limited, rendering them as black box models. To address this issue, numerous methods have been proposed to enhance the explainability and interpretability of ANNs. In this study, we introduce the application of information geometric framework to investigate phase transition-like behaviour during the training of ANNs and relate these transitions to overfitting in certain ANN models.

Information geometry, utilizing the principles of differential geometry, offers a unique perspective on probability and statistics. We define a distance metric based on Fisher information and consider probability distributions as points on a Riemannian manifold. By employing this framework, we quantify the distance travelled by probability density functions (PDFs) as they evolve over time. The concept of information length, representing the total distance traversed on the manifold, allows us to quantify the number of statistically distinguishable states encountered during the time evolution. Within the context of ANNs, we explore how the probability distribution of the parameters of an ANN change during its training, utilizing the information length and information velocity measures. This work contributes to the development of robust tools for improving the explainability and interpretability of ANNs, aiding in our understanding of the variability that the ANN parameters exhibit during its training.

Explicit constructions for chaotic attractors of piecewise-linear maps

Assoc. Prof. David Simpson

Massey University

Abstract:

The concept of an invariant expanding cone has long been used as a means for establishing the presence of chaos in a rigorous manner, but it is perhaps under-appreciated that such cones work brilliantly for piecewise-linear maps. Immediately they show that chaos is robust in families of piecewise-linear maps because the derivative of the map takes finitely many values. They can be constructed explicitly and in some cases so simply that parameter combinations at which the construction fails correspond exactly to bifurcations where chaos is actually lost. More complex constructions based on higher iterates can be done computationally, and this leads to computer-assisted proofs of chaos. The cones also show chaos is robust to smooth perturbations in the pieces of the map and we use this to show that power converters used widely in electronics can operate chaotically over intervals of parameter values immediately beyond border-collision bifurcations.

Critical Gap Size

Mr Ali Beykzadeh

University of New Brunswick

Abstract:

This study explores the persistence of a population within a single patch with hard boundaries, such as a lake, implying that no disperser leaves the habitat. A potential gap, like a fishing zone, divides the population within the patch. We present a method to calculate the maximum size of the gap that does not affect the stability of the population's non-zero steady state. The population's life cycle is modelled by a one-dimensional domain integrodifference equation (IDE). This approach separates the reproduction and dispersal phases in the species' life cycle. We develop an implicit function that is linked to two key aspects: the demography and dispersal parameters of a species, and the total length of the non-reproductive gap. The function establishes a relationship between the dominant eigenvalue and the maximum length of the gap beyond which the population would collapse. We found that when individuals are more likely to settle in the fishing zone, or when they move slower in it and spend more time there, the fishing area must be shorter, and the no-take length must be larger to maintain the population in the lake. The relationship between the reproduction rate of the species in the no-take area and the optimal length of the fishing zone indicates that a longer fishing zone requires a higher reproduction rate in the no-take area to sustain the total population in the lake. This ensures that the no-take sides of the lake can support each other and prevent population collapse.

Spherical Essentially Non-Oscillatory (SENO) Interpolation

Professor Shingyu Leung

Hong Kong University of Science and Technology

Abstract:

We develop two new ideas for interpolation on S^2. In this first part, we will introduce a simple interpolation method named Spherical Interpolation of order n (SIDER-n). The idea generalizes the construction of the Bezier curves developed for R. The second part incorporates the ENO philosophy and develops a new Spherical Essentially Non-Oscillatory (SENO) interpolation method. When the underlying curve on S^2 has kinks or sharp discontinuity in the higher derivatives, our proposed approach can reduce spurious oscillations in the high-order reconstruction. We will give multiple examples to demonstrate the accuracy and effectiveness of the proposed approaches. The work was supported in part by the Hong Kong RGC grant 16302819.

Modelling the mechanism behind the long range alignment of ellipse shaped particles in 2D: from two interacting cells to collective behaviour

Ms Vivienne Leech

University College London

Abstract:

Alignment and self-organisation of particles is a phenomenon observed in various contexts and at different scales in biology. In nature, we see alignment in schools of fish, flocks of birds, and groups of people. Zooming in at a cellular level, we observe alignment in bacterial swarms and the alignment of fibroblasts in tissue.

Fibrotic tissue is associated with numerous pathologies, including cancer, liver disease, and cystic fibrosis, and is believed to contribute to up to 45% of worldwide deaths. One key difference between fibrotic tissue and healthy tissue is that the fibroblasts, the cells which make up the bulk of the tissue, align with each other over a relatively large length scale of up to 6 cell lengths, whereas there is little cell alignment in healthy tissue.

We develop an agent-based modelling framework to model the alignment of self-propelled, interacting ellipse-shaped particles to understand the mechanism behind the long range alignment that is observed in fibrotic tissue. Though there are few known analytical techniques to analyse agent based models, we can analyse the system for two interacting cells with some symmetry imposed. We can then computationally analyse the model for many interacting cells and see whether our analytical results are reflected in our computational findings. Finally, we can compare our model output with experimental data and think about extra mechanisms that could be included to further explain what is observed experimentally.

Understanding the role of geometry and cross-diffusion in pattern formation

Dr Gulsemay Yigit

Bahcesehir University

Abstract:

In this talk, we present analysis of reaction-diffusion systems to understand the role of geometry and linear cross-diffusion. By deriving conditions on the domain length for rectangular, circular and annular geometries, we generate parameter spaces associated with Turing diffusion-driven instability, Hopf and trascritical instabilities. Furthermore, by selecting model parameters from the parameter spaces generated under appropriate geometry and linear cross-diffusion, we are able to demonstrate that linear cross-diffusion coupled with appropriate geometry is able to generate patterns which cannot be obtained through the classical long-range inhibition, and short-range activation mechanism for pattern formation. To support theoretical findings, finite element numerical simulations on rectangular, circular and annular geometries are presented.

Experimental bifurcation analysis of a deformable bubble using control-based continuation

Mr Sammy Ayoubi

University of Manchester

Abstract:

Control-based continuation (CBC) is an experimental method used to explore the bifurcation structure of non-linear systems. Feedback control and continuation techniques are used to discover and stabilize a system's steady states non-invasively, thus allowing them to be observed. To date CBC has been applied to simple mechanical systems such as oscillating pendulums, springs and bending beams. We apply CBC in a spatially extended system for the first time. The system investigated is an air bubble confined in a Hele-Shaw channel filled with silicone oil. The bubble is placed in the centre of a straining flow and is unstable to both translation and deformation. Real-time feedback control is used to achieve bubble steady-states by injecting/withdrawing fluid from the channel, based on the bubble's position and shape. A bifurcation diagram is constructed mapping the bubble's deformation to the injection flow rate of the straining flow. Two unstable solution branches are found which are not observable otherwise.

The role of linear focusing and resonant trapping in generating extreme waves over a submerged sill

Professor Emiliano Renzi

Northumbria University

Abstract:

We explore dispersive wave amplification near a submerged circular sill on a flat seabed, a phenomenon crucial for generating high-amplitude waves, which endanger navigation. Using potential flow theory, we separately solve the velocity potential in the ocean and sill regions. Matching is achieved through integral equations and a Galerkin expansion. Validated against analytical expressions and numerical solutions, our model extends existing theories. For relatively short waves, the sill acts as a wave lens, concentrating energy. Longer wavelengths transition from wave focusing to partial trapping atop the sill. In intermediate water depth, simultaneous focusing and partial trapping lead to extreme wave amplitudes, exceeding the incident wave by up to six times. We suggest that this explains local peaks in skewness and kurtosis near submerged circular shoals, observed in recent simulations.

Deep Learning Methodology for Perron-Frobenius Problems

Mr Tanakorn Udomworarat

University of Nottingham

Abstract:

Invariant densities of Perron-Frobenius operators play an important role in dynamical systems. It provides knowledge of the system's behavior for long-term evolution. Numerical methods are required as it is challenging to find such densities. In this talk, we will propose a novel deep learning methodology, named the Deep Maximum Entropy Method, to approximate the invariant density. Our method minimizes the suitable loss function inspired by the well-known maximum entropy method. Numerical results show better approximations compared with the standard method. Moreover, its data-driven nature allows us to use the Deep Maximum Entropy Method in scenarios where the exact form of a phase map is unknown.

Monotone Travelling waves in the Rosenau-KdV equation

Dr Michael Grinfield

University of Strathclyde

Abstract:

We consider the existence of monotone travelling waves in the generalised Rosenau-KdV equation, which demonstrates phenomena that cannot be seen in the well-known Burgers-KdV equation as the Rosenau-KdV equation exhibits a minimality exchange. Many problems are still open.

This is joint work with N. Bedjaoui and G. Maypaokha (UPJV).

Evaporation of a droplet on a porous substrate

Mr David Craig

University of Strathclyde

Abstract:

Both evaporation and imbibition phenomena play vital roles in a variety of industrial applications, such as inkjet printing, paintwork restoration, and pharmaceutical coatings. However, the majority of the previous research has focused on the evaporation of liquid droplets on solid (i.e. non-porous) substrates. Analytical models, based on lubrication theory, are developed for the evolution of a droplet subject to simultaneous evaporation and imbibition. Specifically, the evolution of a thin droplet situated on an initially dry or an initially flooded porous substrate of varying thickness is analysed. The evaporation from the droplet is driven by the diffusion of liquid molecules into the passive atmosphere. In contrast, the imbibition from the base of the droplet is driven by the pressure difference between the droplet and the porous substrate. We use Darcy's law to model the flow of liquid within the substrate. We analyse the evolution, and hence the lifetime, of such droplets, in a variety of modes, specifically the constant angle, constant radius, stick-slide, and stick-jump modes. While, as expected, imbibition leads to a decrease in the lifetime of the droplet, the dynamics of a droplet undergoing pure imbibition are qualitatively different from those of a droplet undergoing pure evaporation. Furthermore, our results demonstrate that imbibition-driven flow within the droplet is qualitatively different to evaporation-driven flow. In particular, we show that in the limit of a sufficiently small Péclet number the transport rate of particles within the droplet to the contact line is enhanced by the effect of imbibition.

Self-assembly at quasicrystalline surfaces

Dr Sam Coates

University of Liverpool

Abstract:

I will summarise a few key results in the self-assembly of atomic and molecular species at the surfaces of intermetallic quasicrystals. I will highlight any similarities across these systems, the different growth modes that have been observed, and the possible directions for future, similar work. If time permits, I will discuss the possibility of soft matter designer quasicrystals.

Physics-informed Bayesian inference of external potentials in classical density-functional theory

Mr Antonio Malpica-Morales

Imperial College London

Abstract:

We develop a statistical-learning framework to infer the external potential exerted on a classical many-particle system. We combine a Bayesian inference approach with classical density-functional theory (DFT) yielding a probabilistic description to reconstruct the external-potential functional form with inherent uncertainty quantification. Our framework is exemplified with a grand-canonical one-dimensional classical particle ensemble with excluded volume interactions in a confined geometry. The required training dataset is generated using a Monte Carlo (MC) simulation with the external potential applied to the grand-canonical ensemble. The resulting particle coordinates from the MC simulation are fed into the learning process to approximate the external potential. This eventually allows us to characterize the equilibrium density profile of the system by using the DFT apparatus. Our approach benchmarks the inferred density against the exact one calculated through the DFT formulation with the true external potential. The proposed Bayesian procedure accurately infers both the external potential and the density profile. The seemingly simple case study presented in this work has the potential to serve as a model prototype for a wide variety of applications, such as adsorption, wetting, and capillarity.

Quasi-crystalline order in vibrating granular matter

Dr Andrea Plati

Laboratoire de physique des solides (LPS), France

Abstract:

The discovery of quasi-crystals in metallic alloys more than four decades ago has required a full reconsideration of our definition of a crystal structure. After that, quasi-crystalline structures have also been discovered at much larger length scales in different microscopic systems for which the size of the elementary building blocks ranges from the nanometre to the micrometre scale. In my talk, I will report the first experimental observation of spontaneous quasi-crystalline self-assembly at the millimetre scale. This result is obtained in a fully athermal system of macroscopic spherical grains vibrating on a substrate. Starting from an amorphous disordered phase, the grains begin to locally arrange into three types of squared and triangular tiles that eventually align, forming an 8-fold symmetric quasi-crystal that has been predicted in simulation but not yet observed experimentally in non-atomic systems. These results not only demonstrate an alternative route for the spontaneous assembly of quasi-crystals but are of fundamental interest for the connection between equilibrium and non-equilibrium statistical physics.

Reference:

A. Plati, R. Maire, E. Fayen, F. Boulogne, F. Restagno, F. Smallenburg, and G. Foffi

https://arxiv.org/pdf/2307.01643.pdf (accepted in Nature Physics)

Lubrication dynamics of a settling plate

Mr Andrew Wilkinson

Open University, School of Mathematics & Statistics

Abstract:

If a flat, horizontal, plate settles onto a flat surface, it is known that the gap h decreases with time t as a power-law: $h \sim t^{(-1/2)}$.
We consider what happens if the plate is not initially horizontal, and/or the centre of mass is not symmetrically positioned: does one edge contact the surface in finite time, or does the plate approach the horizontal without making contact?
The dynamics of this system is analysed and shown to be remarkably complex. We find that, depending upon its initial position and the position of the centre of force, the plate might either make contact in finite time or settle progressively without ever making contact. If contact is made the plate may then either pivot or slide and we examine conditions giving rise to both outcomes.
Our results show an excellent agreement between analytical exact solutions, asymptotic solutions, and numerical studies of the lubrication equations.

Based on a paper published in the Journal of Fluid Mechanics, Dec 2023. Authors: A Wilkinson, M Pradas, M Wilkinson.

Electrocuting flowers: a guide from AAA to bee

Mr Samuel Harris

University College London

Abstract:

Flowers and pollinators have co-evolved over millennia to produce fascinating sensory properties. One such recent discovery was that bees and spiders can detect natural electrical fields. Thus, our attention turns to flowers. Acting as dielectrics, flowers inductively charge in electrical fields. Considering the source as that of charged pollinators or the Earth's background electrical field, we seek to answer: whether flowers use this to their advantage to become more detectable to pollinators? And how does floral geometry (petal shape and number) affect the perturbed field? To investigate this, the electric field interior and exterior to the flower is modelled numerically. A two-dimensional approximation is taken, and a AAA-least squares method used to find a rational approximation of the harmonic electric potential. Some time is spent examining this method; its application to two-domain problems appears to be new. The algorithm gives accurate and rapid results dependent on only three parameters: the relative permittivity of the flower – its ability to store electrical energy - the petal number and the location of the pollinator(s). The results show that flowers display distinct information about their morphology and pollen levels at distance through the perturbed electric field. Pollinators then detect these signals while also sensing other nearby pollinators. Some arthropods, such as the crab spider, may even use the flower to mask their own presence and draw in unsuspecting prey. Results in 3D are also produced using the COMSOL package and show that the 2D results are a good proxy for the overall 3D behaviour.

The Pfaffian Structure of CFN Phylogenetic Networks

Dr Samuel Martin

Earlham Institute

Abstract:

Algebraic techniques in phylogenetics have been successful at proving identifiability results and have led to novel phylogenetic reconstruction algorithms. In this talk, I will present recent work on a fundamental class of phylogenetic networks under the Cavendar-Farris-Neyman (CFN) evolutionary model. Adopting the approach of algebraic statistics, we use a projective algebraic variety to represent this model and show that an affine open patch containing the stochastic region of the model factors through the space of skew-symmetric matrices via Pfaffians. This enables us to give an explicit Gröbner basis for the associated ideal. Using this, we prove a fundamental identifiability result and develop an algorithm to accurately infer phylogenetic networks from DNA sequence data. Joint work with Joseph Cummings, Elizabeth Gross, Benjamin Hollering, and Ikenna Nometa.

Hybrid modelling for cancer invasion and metastasis

Mr Dimitrios Katsaounis

University of St Andrews

Abstract:

Cancer cells have the ability to interact with the tumour microenvironment and invade the surrounding tissue by reformulating the extracellular matrix (ECM). The coordinated actions of cancer cells, the ECM, cancer associated fibroblasts (CAFs), and the epithelial to mesenchymal transition (EMT) result to in the invasion of the tissue. In this work a multiscale hybrid mathematical model is proposed, which combines the macroscopic nature of the phenomenon, where solid tumours of epithelial-like cancer cells (ECCs) invade the tissue, as well as the microscopic individual based strategies of mesenchymal-like cancer cells (MCCs). The model consists of partial and stochastic differential equations that describe the evolution of the ECCs and the MCCs while accounting for the transitions between the two phenotypes. Numerical simulations of the proposed model capture the heterogenous nature of cancer invasion and metastasis, where we observe simultaneously the growth of the solid body of the tumour and the creation of cancer islands by the MCCs away from the initial location. We extend our numerical simulations to a multi-organ framework where the individual cancer cells, active in a primary tissue, enter the vasculature network and can arrive in different organs and create smaller metastases.

New year, new VisualPDE: fancy features, scientific stories, and more

Dr Benjamin Walker

University of Bath

Abstract:

Last BAMC, we introduced VisualPDE and were overwhelmed by the response from the community for our pet project. Since then, over 20,000 people have interacted with PDEs live in their browser (around 20,000 more than we ever expected) from over 100 countries, performing more than 52 days worth of simulations. Spurred on by this, we've spent the year constantly updating VisualPDE, with new features, new content, and what we hope is a step change in user friendliness. In this talk, we'll showcase some of these new capabilities and how they are changing the way we teach, research, and communicate mathematics. We'll also highlight ways in which VisualPDE reaches beyond the classroom, changing how we can interact with industry and the general public through innovative interactive content. For those of you at BAMC last year, you'll experience déjà vu as we recreate the scientific stories told in the first plenary talk of BAMC23, all interactive and live on your device.

Semi-infinite travelling waves arising in a diffusive Gompertz model with a moving boundary

Dr Nabil Fadai

University of Nottingham

Abstract:

We examine semi-infinite travelling wave solutions of the reaction-diffusion equation, $u_t = u_{xx} - u \log(u)$, with a moving-boundary condition that relates the speed of the moving boundary to the population flux (often called a Stefan-like parameter). Such a choice of reaction term, often called a Gompertz growth term, arises in ordinary differential equation applications in mathematical biology, but rarely in the natural extension to partial differential equations or travelling waves. While previous work of related moving-boundary reaction-diffusion models has demonstrated that such a diffusive Gompertz model admits travelling wave solutions for all positive wavespeeds c, the logarithmic-strength singularity in the gradient of the reaction near u=0, which determines the relationship between the Stefan-like parameter and the wavespeed, has not been previously examined. In this work, we provide asymptotic approximations of the travelling wave profiles and the Stefan-wavespeed relationship in the large wavespeed limit. Using a Liouville-Green (LG) ansatz, we determine how the Stefan-like parameter becomes exponentially large for large wavespeeds. This LG analysis is also used to examine travelling waves arising from the generalised logarithmic growth function family. In all cases, the asymptotic approximations of the travelling waves and Stefan-wavespeed relationships agree with numerical simulations to high accuracy.

Numerical schemes for SIR model

Assistant Professor Canan Akkoyunlu

Istanbul Kültür University

Abstract:

Mathematics has played very important role in humanity because of modelling and simulation for

infectious diseases. The spread of infectious diseases can be modelled and simulated. The SIR model is the most popular epidemic model with three populations, the susceptible group S, infectious group I, and recovered group R. It characterizes infectious diseases that provide immunity upon infection.Unfourtunately, the SIR model does not have an analytical solution for the time course of its popula-tions. A numerical solution for SIR model was done by the use of two different scheme, the rst is the linearly implicit scheme (LIS) and the second is the average vector eld (AVF) method. The numerical experiments are compared with the results obtained by AVF method and LIS.

Modelling symptom propagation in respiratory pathogens

Miss Phoebe Asplin

University of Warwick

Abstract:

Symptom propagation occurs when the symptoms an individual experiences are correlated with the symptoms of the individual who infected them. Symptom propagation may dramatically affect epidemiological outcomes, potentially causing clusters of severe disease or chains of mild infection which generate widespread immunity.

We propose a novel framework for incorporating different levels of symptom propagation into models of infectious disease transmission via a single parameter, α. Varying α tunes the model from having no symptom propagation ($\alpha = 0$) to one where symptoms always propagate ($\alpha = 1$). We apply this framework to a standard SEIR ODE model with two severity classes: mild and severe.

Through analytical and computational exploration, we investigated the effect of α on epidemiological outcomes, including the basic reproduction number (R0) and the proportion of cases that are severe. We also explored how symptom propagation impacted the relative performance of two vaccines with different actions: symptom-attenuating and infection-blocking.

In the absence of interventions, stronger symptom propagation increased R0 (assuming either mild or severe cases were more transmissible). Further, if severe cases were more transmissible, stronger symptom propagation increased the proportion of cases that were severe. We found that the symptom-attenuating vaccine was more effective at reducing prevalence (all infections and severe cases) for higher strengths of symptom propagation. However, for an infection-blocking vaccine, symptom propagation had no impact on effectiveness. In addition, we found parameter values for which determining whether a symptom-attenuating or infection-blocking vaccine would be more effective depended on the strength of symptom propagation.

Stabilisation of falling liquid films with restricted observations

Mr Oscar Holroyd

University of Warwick

Abstract:

We propose a method to stabilise an unstable solution to equations describing the interface of thin liquid films falling under gravity with a finite number of actuators and restricted observations. As for many complex systems, full observation of the system state is challenging in physical settings, so methods able to take this into account are important. The Navier-Stokes equations modelling this interfacial flow are a complex, highly nonlinear set of PDEs, so standard control theoretical results are not applicable. Instead, we chain together a hierarchy of increasingly idealised approximations, developing a control strategy for the simplified model which is shown to be successfully applicable to direct numerical simulations of the full system.

Mechanics of extracellular matrix maintenance in biological tissues

Dr Matthew Butler

University College London

Abstract:

Many biological systems have extra-cellular matrices (ECMs) made from proteins that are used for protection from environmental stresses, as well as a support structure for development and growth. Examples occur in bacterial biofilms and tissue membranes. These solid skeletons are often a major source of mechanical resistance, which are actively maintained by the accompanying living, growing community of cells. Questions remain as to how the bulk mechanical properties of these proteinaceous ECMs depend on this cell maintenance. I will present a spring-based model that aims to capture how the rejuvenation of elastic material in ECMs can give rise to behaviours, such as stress relaxation, that are observed in these systems. From the microscale behaviour, we will investigate the system-scale mechanics and compare our results to reduced effective models.

Revealing 3D opposing vortices through reconstruction of 3D free sperm dynamics

Miss Xiaomeng Ren

University of Bristol

Abstract:

We investigate the dynamics and hydrodynamics of experimental human spermatozoa swimming freely in 3D. We track sperm behaviors in the laboratory frame of reference via imaging microscopy, and extract the first-ever 3D body frame waveform of a free human sperm from the experimental data, capturing the flagellar beating relative to a fixed head. Numerical reconstructions on sperm motility are conducted utilizing the experimental waveform, and the reconstruction accuracy is validated through comparisons with experimental results, which achieve an unprecedented level of accordance between predicted and observed 3D cell trajectories and speeds. Our hydrodynamic analysis reveals a novel average flow pattern, characterized by 3D opposed vortices, along the observed sperm body in an unconfined fluid. This flow structure, distinct from previous studies in the context of sperm vortex, exhibits a dependency on boundary conditions and sperm swimming time period. Notably, our findings demonstrate the reproducibility of such opposite vortices across different sperm waveforms, suggesting a fundamental flow structure for the 3D free-swimming spermatozoa in a bulk fluid.

Principles of a Axisymmetric, Degenerate Rayleigh Wave Resonator

Dr Barry Gallacher

Newcastle University

Abstract:

Surface Acoustic Wave devices are a employed in a vast number of applications ranging from signal processing (filters, oscillators), biosening, gas detection etc. In these devices plane wavefronted Rayleigh waves are employed and as result they are essentially 1 dimensional. Axisymmetric and cyclically symmetric structures can exhibit degenerate dynamic behaviour which offer the attractive feature of common mode rejection of undesirable environmental effects which degrade the device performance. In this paper it is shown that a circular acoustic cavity bounded by a set of circular reflector grooves and formed from elastically isotropic material, can support degenerate resonant behaviour for circular wave-fronted Rayleigh waves. The Rayleigh wave response to a time-harmonic, circular line-load, located within a circular acoustic cavity is determined and expressed in terms of the family of Bessel functions whose order is determined by the cyclic order of the circular line-load. A multiple scales perturbation method is applied to the homogenised equations of motion and boundary conditions. It is shown that significant accumulative reflection of the circular wave-fronted Rayleigh waves from the reflective array can occur thus satisfying the conditions for degenerate resonance. Thus, the resonator has potential application as a Coriolis gyroscope or resonant mass sensor. The perturbation/homogenisation method of analysis shows that the zeroth order solution in the array is affected at first order by periodic irregularities at approximately half the wavelength of the incident Rayleigh wave. The Q-factor of the resonator is calculated and is expressed in terms of the wavenumber defining the reflective array.

Julia sets in relaxed Schröder and Newton-Raphson maps: periodic points, escape points, symmetry-breaking

Dr J. M. Christian, D. Elsby, and S. Alali

University of Salford

Abstract:

The Schröder algorithm is a generalization of the well-known Newton-Raphson (NR) iterative scheme for finding approximate roots of functions [A. S. Househölder, Principles of Numerical Analysis (Dover, New York, 1974)]. In this talk, both methods are deployed on the complex plane to study a simple problem: computing the fourth roots of –1. The root-finding aspect is not of principal concern. Rather, our analysis is from the standpoint of discrete dynamical systems where maps, their higher-order iterates, and their Julia sets (fractals) take centre stage. Ultimately, our interest lies with uncovering the detailed nature of the building blocks that make up the Julia sets for our Schröder and NR maps––periodic orbits. For greater generality, we have introduced a relaxation parameter whose impact on the structure of the periodic orbits can be nontrivial.

We will present some recent results showing how Julia sets for the Schröder and NR maps can be decomposed. Particular emphasis is placed upon the derivation and numerical solution of polynomial equations whose roots are period-M points, where M = 2, 3, 4,… For the NR map, the polynomial degree increases as exp[ln(4)M] while, for the Schröder map, the divergence is an even more severe exp[ln(8)M]. In all cases, the associated periodic orbits are unstable and show extreme sensitivity to arbitrarily-small perturbations. They also break-up into distinct families, each with its own signature and which (through linearization) can be classified according to its instability growth rate. We will conclude by estimating the dimension of the computed Julia sets.

Stratified Tearing Instabilities

Mr Scott Hopper

Newcastle University

Abstract:

The resistive tearing mode is an MHD instability that frequently arises in plasma physics, and may also operate in the Sun's stably stratified tachocline. Previous analytic work has been restricted to either the unstratified regime or the limit of slow growth rate. We have derived a dispersion relation for tearing instability in a Boussinesq fluid, revealing multiple regimes as the strength of the stratification is increased. We find that weak stratification suppresses the instability for large length-scales, whereas moderate stratification tends to suppress smaller length-scales, and strong stratification affects all length-scales. Hence the scale of the fastest growing mode changes non-monotonically as the stratification is increased.

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Ultrasonic sensing of bearings

Mr Matheus de Carvalho Loures

University of Sheffield

Abstract:

Wind turbines, to be cost effective, are becoming larger and larger, with costs in the millions of pounds for just one turbine. Their individual cost and size make it now worthwhile to develop sensing methods that use elastic waves to monitor performance or identify defects. In this talk, we show how to develop a method to predict the forces in rotating components by measuring the emission of elastic waves. Our method combines a modal decomposition, with elements from the theory of classical and statistical inverse problems. Broadly speaking our method is a form of tomography. Tomographic methods often require too many sensors for practical applications. However, with the clever use of symmetries and prior knowledge, we greatly reduce the number of sensors to just three.

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Bounce of a rubber ball and other impact phenomena

Dr Stanislaw Biber

University of Bristol

Abstract:

When a highly elastic rubber ball (known commercially as "Superball") is thrown under a table, it bounces off the ground, the table, and then off the ground again, coming back the same way it entered under the table. This phenomenon can be explained by assuming the conservation of kinetic energy and angular momentum. However, when the coefficient of friction at any stage is changed (for example, by spilling water on the ground), we see the exact opposite effect, in which the ball goes through and leaves the space under the table on the other side.

In this talk, we explore the gaps in the theory of impacts for elastic and rigid bodies and present our approach to bridging these theories. The aim is to model the elastic properties of a rubber ball, while at the same time accounting for the effects of friction. We compare our model with other impact phenomena, arising when, for example, the bounce surface is no longer rigid, but can deform during the impact.

Exact Solution to the Quantum and Classical Dimer Models on the Spectre Aperiodic Monotiling

Ms Shobhna Singh

Cardiff University

Abstract:

The decades-long search for a shape that tiles the plane only aperiodically under translations and rotations recently ended with the discovery of the `spectre' aperiodic monotile. In this setting we study the dimer model, in which dimers are placed along tile edges such that each vertex meets precisely one dimer. The complexity of the tiling combines with the dimer constraint to allow an exact solution to the model. The partition function is Z=2^(N_{Mystic} + 1) where N_Mystic is the number of `Mystic' tiles. We exactly solve the quantum dimer (Rokhsar Kivelson) model in the same setting by identifying an eigenbasis at all interaction strengths V/t. We find that test monomers, once created, can be infinitely separated at zero energy cost for all V/t, constituting a deconfined phase in a 2+1D bipartite quantum dimer model.

Bubble racing in a Hele-Shaw cell

Mr Daniel Booth

University of Oxford

Abstract:

We study theoretically and experimentally the propagation of inviscid bubbles in a Hele-Shaw cell under a uniform background flow, in the regime where each bubble remains approximately circular. New experimental results for the velocity of an isolated bubble are found to agree well with theoretical predictions. For a train of three identical collinear bubbles, we observe that the middle bubble moves faster than the leader, again in agreement with theory. Finally, we examine a system of two non-identical bubbles on different streamlines of the background flow, with the larger one initially behind. We find that the larger bubble overtakes the smaller one, and they avoid contact by rotating around each other while passing.

Steady free-surface waves on an arbitrary distribution of vorticity

Dr Alex Doak

University of Bath

Abstract:

Research of free-surface waves typically assumes irrotational flow. This is a good assumption in many cases, but there are settings where the base flow upon which free-surface waves propagate has shear, such as waves sheared by wind. Some authors have explored the problem of constant vorticity, which has the advantage that there exists a particular solution for the rotational part of the flow. One can then solve for the irrotational correction using methods similar to those in potential flow. The problem of waves on an arbitrary vorticity is more challenging. A number of studies utilize the Dubreil-Jacotin transformation. This maps the flow onto a simple domain in which the free-surface is fixed, but the mapping does not allow for either overhanging streamlines or internal stagnation points (and hence critical layers), both known to be interesting features of waves with constant vorticity.

A recent paper by Wahlén & Weber (2022) reformulated the problem using conformal mappings. This formulation, like the Dubreil-Jacotin mapping, maps the flow to simple geometry, but also allows for waves with both internal stagnation points and overhanging profiles. In this talk, we present a method for discretising and numerically solving this system. Irrotational and constant vorticity waves are used as a test case, and solutions with arbitrary vorticity are presented. The talk finishes with a possible extension to stratified flows with a free-surface.

Mathematical modelling of active fluids in a confined rectangular region

Mr Ijuptil Joseph Kwajighu

University of Glasgow

Abstract:

Active fluids are systems composed of self-driven units that consume and convert energy into directed motion. Motivated by possible applications in the design of sensors and for the general understanding of various biological processes, we present a 2D theoretical and computational study involving the mathematical modelling of active fluids confined between two parallel plates using an adapted form of the Ericksen-Leslie equations. We numerically solve the model, with imposed hybrid–aligned nematic (HAN) anchoring – where the average orientation of the self-driven units is parallel to the substrate at one plate and perpendicular to the substrate at the other. We characterise the range of flow profiles for varying magnitudes of the activity at a fixed channel width. For low-magnitude activities, the flow is characterised by a positive (negative) gradient of the velocity field in left-hand (right-hand) regions. Further increasing the magnitude of the activities results in more localised flow. However, the system exhibits spatial fluctuation for sufficiently high magnitudes of activities, although there are no time-dependent oscillations in the system. For lower activities, we observed unidirectional flow, while for high activities we observed bidirectional flow characterised by an antisymmetric distorted state at the middle of the region. Our results lead to potential applications for designing sensors owing to the ability of small changes in the alignment of the self-driven units to ensure long-range effects in the orientation and flow of the active fluid.

Does an intermittent dynamical system remain chaotic after drilling in a hole?

Mr Samuel Brevitt

Queen Mary University of London

Abstract:

Chaotic dynamical systems may be characterised by a positive Lyapunov exponent, which measures the exponential rate of separation of nearby trajectories. However in a wide range of so-called weakly chaotic systems, the separation of nearby trajectories is sub-exponential, for example stretched exponential, in time; and therefore in such cases the Lyapunov exponent vanishes. When a hole is introduced in chaotic systems, the Lyapunov exponent on the system's fractal repeller can be related to the generation of entropy and the escape rate from the system via the escape rate formalism, but no suitable generalisation exists to weakly chaotic systems. In this work we show that in a paradigmatic one-dimensional weakly chaotic iterated map, the Pomeau-Manneville map, the generation of generalised Lyapunov stretching is completely suppressed in the presence of a hole. These results are shown based on numerical evidence, and explained with a fully analytic stochastic model.

Restricted Adaptive Probability-Based Latin Hypercube Design

Dr Huijuan Li

University of Warwick

Abstract:

The complexity of environmental sampling comes from the combination of varied inclusion probabilities, irregular sampling region, spatial-filling requirements and sampling cost constraints. This article proposes a restricted adaptive probability-based latin hypercube design for environmental sampling. Meriting from a first stage pilot design, the approach largely reduces the computation burden under traditional adaptive sampling without network replacement, while still achieves the same effective control on the final sample size. Under the restricted adaptive probability-based latin hypercube design, Thompson-Horvitz and Hansen-Hurwitz type estimators are biased. A modified Murthy-type unbiased estimator with Rao-Blackwell improvements are thus proposed. The proposed approach is shown to have better performances than several well-known sampling methodologies.

A continuum theory for odd rods

Dr Sami Al-Izzi

University of Oslo

Abstract:

Living systems are chiral on multiple scales, from constituent biopolymers to large scale morphology, and their active mechanics is both driven by chiral components and serves to generate chiral morphologies. Increasingly such active chiral phenomena are being viewed through the lens of odd mechanics in both biological systems and in bio-inspired soft robotics.

Motivated by recent experiments in bio-inspired soft robotics, we derive a continuum theory of elastic rods with odd contributions to the bending moment/stress tensor. This leads to unstable waves when inertia is non-negligible. We discuss the dispersion relations, coarsening dynamics and how these theories relate to experiments on odd robotic filaments and discrete simulations of such systems.

Multiscale Modelling of vascular tumours subjected to electrophoresis anticancer therapies

Miss Zita Borbala Fulop

University of Glasgow

Abstract:

Electrophoresis facilitated cancer treatment has demonstrated experimental efficacy in enhancing drug delivery within vascularised tumours. However, the lack of realistic mathematical models with direct measurements in the context of electrochemotherapy poses a challenge. We investigate the impact of an applied electric potential on the flow of Darcian-type fluid occurring in two distinct phases: the tumour and healthy regions. We employ the asymptotic homogenisation technique, assuming that the macroscale of the tumour domain is larger than the microscale characterised by vessel heterogeneities. We retain information about the microstructure by encoding information in the homogenised coefficients. We take into account both vascularisation and the microscale variations of the leading order and fine scale electric potential. The resulting effective differential problem reads as a Darcy-type system of PDEs, where the flow is driven by an effective source. The novel model can be used to predict the effect of an applied electric field on cancerous biological tissues, paving a new way of improving current electrochemotherapy protocols.

When reservoir computing meets information theory

Mr Zonglun Li

University College London

Abstract:

Brain can be seen as a giant reservoir composed of neurons and other cell types. It is widely believed that numerous learning mechanisms are taking place simultaneously through neuronal interactions in order to modulate synaptic weights which can potentially regulate information processing. However, it is still largely unclear how external stimuli may affect synaptic connections and therefore, information storage and transmission. Spike Timing Dependent Plasticity (STDP) is a form of biologically plausible rules enabled by the temporal difference between spike arrival times of pre- and postsynaptic neurons. It is reckoned to facilitate learning and information storage in the brain, as well as the formation of neuronal circuits during brain development. In addition, the Shannon entropy is a pivotal measure in information theory to quantify the amount of information (uncertainty) of complex systems. Hence, in this work, we employed a mathematical model to study how external stimuli may alter the information level through STDP rules.

Understanding linguistic dynamics in agent-based communal networks

Miss Emily Claughton

University of Nottingham

Abstract:

Linguistic variation has been well studied on an individual basis with several accepted mechanisms as to why individuals change the way that they speak and the vocabulary they use. However, the answer to the question "What makes populations adopt new language on a communal level?" remains largely unclear. Lack of data on individual dialects and the scale of populations has traditionally rendered this problem difficult to solve. Mathematical modelling is ideally suited to explore potential mechanisms of this population phenomenon. Representing a community of speakers as an interconnected network, we demonstrate that agent-based network modelling can help us understand linguistic change in networks and allows us to investigate it as a function of network parameters.

As an application, we focus on England in the Middle Ages. Our novel representations of data from the Linguistic Atlas of Late Medieval English reveal distinct and stable patterns of word use, and we propose a mechanism within a network model that could account for them.

Modelling solute transport in the plant phloem

Dr Jacob Jepson

University of Nottingham

Abstract:

In plants, the phloem facilitates the efficient transport of nutrients, such as sugars and hormones, from leaves to other tissues, enabling growth and metabolism. In this talk, we develop and analyse a continuum model that describes sucrose transport in the phloem of a seedling. We model the phloem as a cylindrical pipe, and consider the evolution of a viscous sap in which a concentration of sucrose is diluted. Via the pressure-flow hypothesis, sucrose can travel from a source region via the phloem toward a sink region where it is unloaded. We exploit the geometry of the phloem pipe and the laminar nature of the sap flow to reduce the model to two partial differential equations governing the temporal-axial sap pressure and sucrose concentration. Using numerical solutions of this reduced model, we characterise how varying phloem membrane properties, and environmental conditions such as drought, affect sucrose transport and delivery to the sink regions.

Hydrodynamic efficiency limit on a Marangoni surfer

Dr Abdallah Daddi-Moussa-Ider

The Open University

Abstract:

A Marangoni surfer refers to an object positioned within a gas-liquid interface, propelled by gradients in surface tension. In this study, we establish an analytical theorem outlining the lower limit of viscous dissipation by a Marangoni surfer under conditions of small Reynolds and capillary numbers. The minimum dissipation is expressed by the reciprocal difference in drag coefficients between two passive bodies of identical shape to the Marangoni surfer. One body is positioned in a force-free interface, while the other is in an interface with surface incompressibility. The resulting flow is a superposition of both solutions. For a surfer adopting the form of a thin circular disk, the Lighthill efficiency amounts to 1/3. This positioning establishes Marangoni surfers among the most hydrodynamically efficient microswimmers.

Self-assembly phase-behaviour of core-shell particles

Professor Andy Archer

Loughborough University

Abstract:

We investigate the phase ordering (pattern formation) of systems of two-dimensional core-shell particles using density functional theory (DFT) and Monte-Carlo (MC) computer simulations. The particles interact via a pair potential having a hard-core and a repulsive square shoulder. In DFT, the hard-core part of the potential is treated using either fundamental measure theory or a simple local density approximation is used, whereas the soft shoulder is treated using the random phase approximation. The different DFTs are bench-marked using large-scale grand-canonical-MC and Gibbs-ensemble-MC simulations, demonstrating their predictive capabilities and shortcomings. We find that having the dispersion relation is sufficient to identify the Fourier modes governing both the liquid and solid phases. The dispersion relation is the growth/decay rate of density modes as a function of wavenumber k perturbing the uniform liquid state and is also a coefficient in the free energy functional of the term quadratic in the density. Based on this observation, we identify a simple criterion for determining where in the phase diagram the multiple different crystalline (patterned) states occur and identify the wave vectors playing a role in the phase self-assembly. Our approach allows to navigate the very rich phase diagram of core-shell particles, which exhibits a plethora of structures constructed by the coupling density modes which are either entropic in origin, coming from the hard-core (excluded volume) part of the pair potential, or energetic in origin, from the soft-shoulder contribution.

Modelling the impact of climate change on cocoa farming in Nigeria

Professor Chris Budd

University of Bath

Abstract:

Cocoa is an important crop that is predominantly grown in the western part of Africa. However, there have been fluctuations and declining trends in production and several factors have been identified to be responsible for this. A significant factor is the effect of climate variation which could result in a low farm-level yield. Therefore, to understand the contribution of climate variability on the farm-level yield, we construct and analyse a time-delayed model to capture the effect of rainfall on cocoa production. This work uses a system of differential equations to model the crop transition from the flowering stage to pod formation, pod ripening, and then to harvesting. We introduce a periodic forcing function into the model of flowering to account for the impact of seasonal rainfall variations. This leads to a novel nonlinear parametrically forced ODE for the flowering with periodically varying coefficients which is coupled to a time-delayed model for the ripened pod formation and then harvesting. We perform an analysis of all parts of the system proving that it has a periodic solution when (parametrically) forced periodically and we then conduct an asymptotic analysis on this periodic solution to show how its rich behaviour depends on the parameters of the climatic forcing in the model.

Data-Driven decisions in real-time: Can we control an epidemic with uncertainty in infection incidence data?

Dr Sandor Beregi

Imperial College London

Abstract:

Noise and uncertainty in epidemic data pose significant challenges for controlling infectious diseases in real-time. Practical reporting delays and under-ascertainment of infections can mean that feedback-control strategies, which dynamically update control actions (e.g., the choice of when to impose or relax lockdowns) can fail to reduce transmission effectively, overshooting healthcare capacities at substantial cost. As a result, recent studies have proposed simplistic or more conservative approaches that are less sensitive to data quality e.g., interventions triggered by pre-set thresholds or switched at predetermined times. However, an overly conservative strategy may also come at high cost, motivating the study of cost-optimal epidemic control strategies.

We study feedback-control of an epidemic outbreak through non-pharmaceutical interventions (NPIs). We apply model predictive control to develop real-time algorithms for balancing the costs of NPIs against those generated from prevalent infections. We derive metrics for control effectiveness to assess how surveillance noise and uncertainty limit the controllability of an epidemic and explore when choosing feedback control over simpler strategies is beneficial.

Our study demonstrates that, within the limitations of data quality, feedback-based control is viable for epidemic management that can guide efficient resource allocation and that the deteriorating effect of reporting delay and incomplete case ascertainment is no better for simpler, threshold-based policies. Our results confirm that data uncertainty is a significant factor in epidemic control and that reducing or correcting for delay and under-reporting in the surveillance is paramount to improving pandemic preparedness when data-informed, cost-optimal feedback control is applied .

Ocean waves modelled by the forced/damped nonlinear Schrödinger equation

Mr Ben Humphries

The University of East Anglia

Abstract:

The evolution of ocean waves is well modelled by the ubiquitous nonlinear Schrödinger equation (NLS) in the presence of frequency-dependent viscous-like attenuation. In this work we expand on previous studies by presenting a forced/damped NLS to concurrently model the effect of dissipation (e.g. sea ice) and source terms (e.g. wind). We anticipate that this work may motivate analogous studies in experimental wave systems subject to forcing and damping.

Acoustic Sensing of Particulate Material through Layers

Mr Paulo Sergio Piva

University of Sheffield

Abstract:

Most products in chemical engineering and pharmaceuticals involve powder or particulate materials at some stage. Currently, there is no way to constantly monitor or measure the particles in the particulates. To develop a sensing method, we can use acoustic waves transmitted through these materials. In this talk, I will explain how to model waves propagating in a random particulate material by using techniques from statistical mechanics such as ensemble averaging. The model is developed for a broad range of frequencies, which is needed to measure the size of a broad range of particles. The problem of layers involved an unsolved technical challenge, for which now we have a solution.

Economic Nowcasting

Dr Lingyi Yang

University of Oxford

Abstract:

Economic nowcasting refers to the inference of the current state of the economy. The nowcasting literature has arisen to address the need for policymakers to have fast, reliable estimates of key economic indicators, which are often published with a significant delay of over a month. The path signature is a mathematical object which can uniquely identify and capture geometric properties of sequential data; it naturally handles missing data from mixed frequency and/or irregular sampling – issues often encountered when merging multiple data sources – by embedding the observed data in continuous time. We apply regression on signatures for nowcasting, a simple linear model on these nonlinear objects that we show subsumes the popular Kalman filter. We further provide theoretical justification that regression on signatures retains the consistency properties we would expect from ordinary least squares. To demonstrate its performance, we apply this method to both simulated data and real-world data like US GDP growth. In the latter, we achieve a lower root-mean-square error over the evaluation period compared with a dynamic factor model based on the New York Fed staff nowcast model.

Feature Selection for Time Series Forecasting: From the Sea's Depths to Space (and beyond)

Mr Gianluca Audone

Politecnico di Torino - University of Bath

Abstract:

Feature selection plays a crucial role in prediction algorithms. The aim of this talk is to build a robust framework and give a set of tools to conduct the exploratory data analysis needed when dealing with Time Series. The starting point will be ergodic stationary processes through which we rigorously define time series. We introduce the methodologies for trend identification and seasonality estimation to then focus on the insights given from auto-correlation and cross-correlation analyses which help understanding the interplay and dependencies among different features.

Using data from the Comprehensive Nuclear Test Ban Treaty Organization (CTBTO) and space weather data we show some result given by the introduced framework. CTBTO operates a global International Monitoring System, with 11 hydroacoustic stations around the globe located in the deep-sea sound channel. Continuous measurements provide up to 20 years of sound pressures at frequencies of up to 100 Hz. These relatively long timescales allow investigating the effects of climate over that period. The solar activity recorded over 14 years consisting of solar wind, geomagnetic and energetic indices measured in situ L1 and, magnetospheric ring current intensity measured on Earth.

This presentation offers insights into the intricate realm of climate studies and space weather prediction, showcasing the significance of meticulous data exploration and feature selection in harnessing the predictive power of time series data.

Predicting the incidence of catheter-associated bacteriuria

Dr Freya Bull

University College London

Abstract:

Urinary catheters (thin tubes used to drain the bladder) are prone to colonisation by bacteria. When this leads to symptoms such as fever, pain, or inflammation, it is known as catheter associated urinary tract infection (CAUTI). CAUTI constitute up to 40% of hospital acquired infections, and the potential of novel materials and coatings to reduce the incidence of CAUTI has attracted much attention; however none of these design changes have been found to be effective, with patient studies finding mixed or limited effects.

Here we apply a simple biophysical model for bacterial colonisation of urinary catheters to predict the incidence of bacteriuria (the presence of bacteria in the urine) in a clinical trial of antimicrobial catheter coatings -- the CATHETER trial. We find that the urethral length determines the timescale of bacteriuria development, and hence that antimicrobial coatings have greater efficacy in preventing bacteriuria in males undergoing short-duration catheterisation than in females.

Nonlinear acoustics in a general 3D duct

Mr Freddie Jensen

University of Warwick

Abstract:

We present a model for weakly nonlinear acoustics in a 3D duct of arbitrary shape, with an application to the physics of brass instruments. This extends the work of Fernando (2011) and Félix (2002) by combining shock physics with complicated geometry. The Euler equations are perturbed about a state of rest, and expanded up to second-order in the perturbation Mach number. The acoustic perturbations are then doubly expanded, first as a Fourier series in time and then in a spatial functional basis deriving from the modes of a straight, cylindrical duct. We are then left with a coupled, countably-infinite set of ODEs for the coefficients of the double expansion. These ODEs must be solved with consideration of the duct outlet physics, so we introduce the admittance, along with its nonlinear correction term (McTavish, 2019), allowing us to prescribe a radiation condition. We show results obtained from this model, and present further work that poses an interesting question about the correct way to define a reflected wave in a complicated duct geometry.

Generalized Wiener-Hopf method for the problem of diffraction by a discrete wedge

Mr Andrey Korolkov

University of Manchester

Abstract:

A problem of wave propagation on 2D discrete lattice is considered. The lattice bears a discrete Helmholtz equation with a 5-point stencil. The first quadrant of the lattice is blocked by setting the field equal to zero there. The problem of diffraction of an incident plane wave by the blocked angle is studied. A new formalism has been developed for this problem. Using a discrete analogue of second Green's identity a functional equation is derived for the problem. Using the symmetries of the problem it is shown that unknown spectral function is meromorphic on the dispersion surface of the lattice, which is a torus. It is shown that such a function can be built in a constructive way using the theory of algebraic functions. Also, it is demonstrated that the functional problem can be reduced to the generalised Wiener-Hopf equation by an appropriate change of variables.

Evolution of an annular viscous tube with variable surface tension

Mr Matthew Shirley

University of Oxford

Abstract:

Bow-tie fibres are a type of specialist optical fibre capable of maintaining the polarisation of light transmitted along them over vast distances, making them useful for applications such sensors and gyroscopes. This property of bow-tie fibres is achieved by carefully oriented mechanical stresses created by regions of glass dopped with additives. The shape and position of these dopped regions can affect the performance and quality of the fibre, motivating us to study how these can be controlled in the manufacturing process.

One of the major steps in the manufacture of bow-tie fibre is the production of a `preform' a solid glass rod, in which the dopped regions of glass are embedded such that the preforms cross-section matches that desired in the final fibre. The preform is produced by depositing thin layers of doped glass on the inner surface of a pure silica tube, and heating the tube up so that it contracts and collapses to a solid rod.

In this talk we will present a model for the initial stage of the process of collapse, modelling the glass tube as a viscous fluid, and the thin doped regions as a variation in surface tension on the inner surface. We will explore how changes to material properties alter the shape formed after the initial collapse, then demonstrate how this process can be controlled by varying the pressure inside the tube and the initial distribution of the doped glass.

Phonon signatures in photon correlations

Mr Ben Humphries

The University of East Anglia

Abstract:

We show that the second-order, two-time correlation functions for phonons and photons emitted from a vibronic molecule in a thermal bath result in bunching and anti-bunching (a purely quantum effect), respectively. Signatures relating to phonon exchange with the environment are revealed in photon-photon correlations. We demonstrate that cross-correlation functions have a strong dependence on the order of detection giving insight into how phonon dynamics influences the emission of light. This work offers new opportunities to investigate quantum effects in condensed-phase molecular systems.

Laplace-transformed stochastic dynamics and the Kramers problem

Dr Steve Fitzgerald

University of Leeds

Abstract:

Transition probabilities for stochastic systems can be expressed in terms of a functional integral over paths taken by the system. Evaluating the integral by the saddle point method in the weak-noise limit leads to a remarkable mapping between dominant stochastic paths and conservative, Hamiltonian mechanics in an effective potential. The conserved "energy" in the effective system has dimensions of power. We show that this power, H, can be identified with the Laplace parameter of the time-transformed dynamics. This identification leads to insights into the non-equilibrium behaviour of the system. Moreover, it facilitates the explicit summation over families of trajectories, which is far harder in the time domain. We apply these ideas to the ubiquitous problem of noise-driven escape over an energy barrier.

Order-disorder criticality in infinite particle chains at zero temperature

Dr Gyula Toth

Loughborough University

Abstract:

The theoretical understanding of the structure of amorphous solids and the nature of the glass transition are among the most actively studied problems in Condensed Matter Physics. In this talk, first we briefly summarise recent results obtained in the framework of the Random First Order Transition (RFOT) theory, which is followed by proposing an improved theory which is based on the randomisation of the crystalline state and the concentration of measure phenomenon. The generic concept is then applied to the infinite Lennard-Jones chain at zero temperature, where the amorphous configurations are generated by sampling a correlated Wiener process. We provide evidence for the existence of an order-disorder critical point located in the metastable crystal regime, and demonstrate the concentration of the amorphous energy density and the resultant force (per particle) around their expectation as a response to fine tuning the correlation function of the randomisation process. The adaptation of the methodology (together with the challenges) for the Classical Density Functional Theory is also outlined.

The role of an incoherent feed forward motif in decoding oscillatory protein expression dynamics in the developing pancreas

Dr Andrew Rowntree

University of Manchester

Abstract:

Ultradian oscillations of transcription factors are known to have functional importance to many cellular processes, however how the periodic expression of such genes is decoded downstream and used in decision making remains unclear. Neurogenin-3 (NGN3) is a transcription factor critical for pancreatic endocrine development.

Using an endogenous knock-in reporter, we show that NGN3 protein oscillates with a 13-hour periodicity in human iPS-derived endocrine progenitors and that the dynamics are linked with controlling the timing of endocrine differentiation. Specifically, differentiation is found to occur sooner when NGN3 exhibits fewer oscillatory pulses with greater peak-to-trough fold-changes; yet how a cell reads these dynamics in decision-making was not clear.

Mathematical modelling using delay differential equations and an incoherent feedforward loop (IFFL) gene motif, we provide an explanation to both the temporal pattern of normal differentiation as well as precocious differentiation. Since certain IFFLs have been known to include "fold-change" detectors as components, we utilise this feature, providing oscillatory signals as inputs to tease out specific downstream dynamics.

Our synthetic findings suggest that oscillatory NGN3 dynamics are decoded via an IFFL by more than one target gene, all of which play important roles in how the timing of differentiation of pancreatic endocrine progenitors is controlled.

References:
Miller, A., Biga, V., Rowntree, A., Chhatriwala, M., Woods, F., Marinopoulou, E., Kamath, A., Vallier, L. and Papalopulu, N., NGN3 Oscillatory Expression Controls the Timing of Human Pancreatic Endocrine Differentiation. Available at SSRN 4658844.BioRxiv: doi: https://doi.org/10.1101/2024.01.10.574974

The 65th British Applied Mathematics Colloquium

Impacts of Liquid Drops: When Do Gas Microfilms Prevent Merging?

Mr Peter Lewin-Jones

University of Warwick

Abstract:

Collisions and impacts of drops are critical to numerous processes, including raindrop formation, inkjet printing, food manufacturing and spray cooling. For drop-drop collisions, increasing the relative speed leads to multiple transitions: from merging to bouncing and then back to merging - transitions which were recently discovered to be sensitive to the drops' radii as well as the ambient gas pressure. The outcome of a drop impacting a liquid bath is even more complex: for a fixed speed, the result can go from merging to bouncing to merging and back to bouncing with increasing bath depth.

To provide new insight into the physical mechanisms involved and as an important predictive tool, we have developed a novel, open-source computational model for both drop-drop and drop-bath events, using the finite element package oomph-lib. This uses a lubrication framework for the gas film and incorporates fully, for the first time, the crucial micro- and nano-scale influences of gas kinetic effects and disjoining pressure.

Our simulations show strong agreement with experiments for the transitions between merging and bouncing, but can also go beyond these regimes to make new experimentally-verifiable predictions. We will show how our model enables us to explore the parameter space and discover the regimes of contact (that are inaccessible to experiments). Finally, we will overview potential extensions to the computational model, including impacts in Leidenfrost conditions and post-contact dynamics.

Modelling the regulation of chronic wounds by tissue inhibitors of matrix metalloproteinases

Miss Sonia Dari

University of Nottingham

Abstract:

Understanding the biochemistry and pharmacology that underpins chronic wounds and wound healing is of high importance as there are over 2 million people in the UK suffering from chronic wounds. Chronic wounds are susceptible to high levels of Matrix Metalloproteinases (MMPs), which are responsible for the modification and proliferation of healthy tissue. High concentrations of MMPs however cease to be beneficial and can lead to the destruction of the healthy tissue. Tissue inhibitors of MMPs (TIMPs) are produced in response to MMPs and are responsible for the regulation of MMP concentrations and thus the development of chronic wounds.

In this talk, we propose a mathematical model that focuses on the interaction of dermal cells, MMPs and TIMPs in the healing process using a system of partial differential equations. Using a parameter set corresponding to healthy biological functioning, we observe the emergence of travelling waves corresponding to a front of healthy cells invading a wound. From the arising travelling wave analysis, we observe that deregulated apoptosis results in the emergence of chronic wounds characterised by a hysteresis effect when apoptotic rates are varied. We observe that this hysteresis effect disappears when TIMP production is increased, providing insights into the role of TIMPs as a regulator of healing. In extending our wound to a two-dimensional spatial domain, an influx of TIMPs to a chronic wound by means of a Robin boundary condition results in the complete healing of a chronic wound, providing further insight into the management of chronic wounds.

How do bee's smell? The multi-physics of honeybee olfaction

Dr Ryan Palmer

University of Bristol

Abstract:

"The sense of olfaction (smell) in honeybee's is largely understood to occur through sensory receptors along their antennae. We study one type of sensor called a placode, which densely covers each antenna in a regular formation. Sitting close the antennae's surface, each placode is covered by hundreds of innervated pores that capture olfactory particles. We seek to understand how the morphology and configuration of placodes along the honeybee's antenna affect the flow of air and how this fluid-structure interaction impacts a bee's ability to smell.

Currently, the precise role of the placode's morphology in olfaction is unknown. Two candidate shapes have been identified which we shall examine. Each is of distinct and unusual morphology presenting as either a pit (with an initial sharp ring and smooth inner dimple) or a mound (with a small divot on top).

Based on the evidence to date, we will model the fluid-structure interaction of the airflow and placode's considering their morphology and configuration to assess their role in volatile capture. Due to the depth and relative length of the placodes to the antennae, the so-called condensed flow equations apply. Initially, we consider two and three-dimensional configurations for a single placode to investigate the local influences of its morphology on the fluid flow. We later extend this scenario to that of a three-dimensional periodic case, whereby the streamwise and crosswise interactions of many placodes is considered. Finally, we compare these results to FEM models and assess the potential role of electrostatics in this olfactory process.

Non-Local Variable-Order Modelling of Avascular Tumor Growth Dynamics

Ms Mariam Al Mudarra

University of Glasgow

Abstract:

In this investigation, we explore the growth of an avascular tumour, focusing on the mass exchange among its constituents and the resultant inelastic distortions from the growth process. A key aspect of the study is the analysis of non-local diffusion, a significant factor which we hypothesise to arise due to the tumour's complex and varied microenvironment. This diffusion is further emphasized by a variable-order fractional operator, highlighting areas within the tissue where the diffusivity of nutrients can be limited. The research also aims to develop a law governing the evolution of growth-induced inelastic distortions. This goal is achieved by identifying generalised forces corresponding to specific kinematic descriptors linked to the growth tensor. The development of this law involves an analysis of the dissipation inequality, revealing the interplay between inelastic distortions and the source/sink terms in the mass balance laws. To comprehend the dynamics of tumour growth and its response to this modelling approach, the study initially assesses the impact of the newly established growth law on the variables driving the tissue evolution. Subsequently, the influence of limited nutrient availability within the tumour, encapsulated by a variable-order fractional operator, on its growth is examined. This comprehensive approach provides valuable insights into the mechanisms of tumour growth and highlights the significant role of nutrient availability.

Electrohydrodynamic interactions of a pair of leaky dielectric droplets

Mr Michael McDougall

University of Strathclyde

Abstract:

When a weakly conducting (leaky dielectric) droplet is suspended in another fluid and exposed to a uniform DC electric field, it becomes polarised and electric stresses tangential to the droplet interface drive fluid motion inside and outside the droplet. In the presence of a second identical droplet, the dynamics of the first droplet are modified due to electrohydrodynamic interactions; most importantly, the droplets start translating due to dielectrophoretic forces and hydrodynamic interactions. A three-dimensional small deformation theory for a pair of widely separated, leaky dielectric droplets suspended in a weakly conducting fluid medium is presented, valid in the limit of high droplet viscosity and surface tension, such that the drop remains nearly spherical. The theory is constructed in accordance with the Taylor-Melcher leaky dielectric model, while retaining the effects of transient charge relaxation and convection. The latter allows for the possibility of drop rotation in sufficiently strong fields. The flow field around each droplet is governed by the Stokes equations. Interfacial Maxwell stresses arise on the surface of the droplets due to the applied field as well as the polarisation of both droplets. Hydrodynamic stresses arise in response to these electric stresses, which must also balance capillary stresses. Further, an analytical expression for the critical electric field strength for rotation of an isolated spherical droplet is derived.

Capillary Bridge Formation between Lipid Membranes by Biomolecular Condensates

Professor Halim Kusumaatmaja

Durham University

Abstract:

Biomolecular condensates, typically formed via cellular phase separation processes, are increasingly recognised to play important roles in cellular organisation. In this work we use a combination of mathematical modelling, live-cell observations, and in vitro studies to investigate the formation of capillary bridges between lipid membranes by biomolecular condensates. Depending on membrane mechanics, geometries, and condensate-membrane interactions, we identify three possible morphologies of condensate capillary bridges; two of which are exclusively observed between highly deformable membranes. Each morphology has distinct mechanical behaviours, and we suggest these can lead to different cellular functions, such as to bring lipid membranes into contact relevant for the formation of membrane contact sites and to enable vesicle-vesicle encapsulation. Our findings unveil previously unrecognized roles of biomolecular condensates and exemplify how condensate wetting on membranes can contribute to intracellular organization.

A moving-boundary approach to controlled drug release with finite dissolution rate

Mr Maniru Ibrahim

University of Limerick, Ireland

Abstract:

We revisit an earlier one-dimensional model for controlled drug release with finite dissolution rate from a polymer matrix. Although the model was originally given in the form of two time-dependent partial differential equations – one, for the concentration of dissolved drug, of reaction-diffusion type and the other, for the concentration of solid drug, with reaction but no diffusion – and then later reformulated into a single effective evolution equation, here we show how the problem can be posed in three distinct stages. In the first stage, solid and dissolved drug are present throughout the layer. In the second stage, dissolved drug is present everywhere, but there is no solid drug in part of the layer. In the third stage, the dissolved drug is again present everywhere, but there is no solid drug anywhere. Consequently, the first and third stages constitute fixed-boundary problems, whereas the second stage is a moving-boundary problem. The model equations are solved numerically in three different ways using the finite-element software Comsol Multiphysics, with the new contribution being the use of boundary immobilization to treat the second stage. Although the three approaches give good agreement, only the new approach is able to give detailed quantitative information about the moving-boundary behaviour at its inception and extinction.

Microfibre Filtration in Washing Machines

Mr Torin Fastnedge

University of Oxford

Abstract:

Microplastic fibres from our clothes make up around 35% of all of the microplastics in our oceans, and it is estimated that each person in the UK produces on average 243g of microplastic fibres per year when washing their clothes in a standard washing machine. Thus, we need to find ways to reduce the number of microplastics, in the form of microfibres, that are drained with the waste water during laundry. Working in collaboration with Beko plc, we are exploring simple mathematical models to capture microfibre shedding and filtration within a washing machine. In this talk, we focus on exploring the concept of removing these microfibres from dirty fibrous water using ricochet separation, a solid-fluid separation technique used by manta ray fish to separate plankton from water. Our aim is to understand the interplay between fluid mechanics, continuum mechanics and clothing materials in order to maximise filtration efficiency, within a washing machine.

Existence of liquid toroids

Mr Kraig Wymer-Webb

University of East Anglia

Abstract:

Cylindrical columns of fluid are known to rapidly break down into droplets as a result of surface tension. However, if the fluid comprising the column is a magnetisable ferrofluid, then this can be prevented. Ferrofluids become magnetised in the presence of an external magnetic field, such as that produced by a current-carrying wire. The force associated with magnetisation opposes surface tension and can stabilise the column.

Toroidal volumes of fluid are similarly known to break down into droplets due to surface tension. In fact, surface tension inhibits such configurations from even being in equilibrium. Taking motivation from the case of the cylindrical column, we will investigate the possibility that a static toroidal volume of ferrofluid surrounding a circular loop of current-carrying wire can exist in equilibrium.

We will derive a boundary-integral equation for the magnetic field and a dynamic boundary condition to be satisfied at the ferrofluid free surface. Equilibrium states of the system, characterised by physical parameters in the problem such as the fluid volume and a magnetic Bond number, are obtained numerically. Validation of the numerical results is achieved via comparison with an asymptotic solution obtained in the limit of the wire-loop's radius becoming large.

A numerical algorithm for a coupled hyperbolic Goursat-Cauchy boundary value problem

Dr Mihaela-Cristina Drignei

University of Pittsburgh at Bradford

Abstract:

A boundary value problem consisting of two hyperbolic partial differential equations (PDEs) coupled by an unknown joining function, and three sets of boundary conditions (Goursat, Cauchy, vertex) will be presented. This boundary value problem is solved for a quadruple: the two functions of two variables involved in the above mentioned PDEs, the unknown joining function of one variable, and one real-valued constant. The numerical algorithm solving this boundary value problem will be discussed, and two examples on the algorithm performance will be presented. This boundary value problem finds applicability to inverse Sturm-Liouville problems.

Turing and wave instabilities in reaction-diffusion systems with cross-diffusion

Mr Edgardo Villar-Sepúlveda

University of Bristol

Abstract:

Turing instabilities have been a big research topic in the last few years, after Alan Turing discovered that, under certain conditions, a stabilizing process can destabilize a system. In a reaction-diffusion system with two components and a diagonal diffusion matrix, the conditions for these instabilities to appear are simple and well-known. However, these are restricted only to steady heterogeneous patterns. In the last few years, they have been generalized to study Turing and Turing-wave instabilities in general n-component reaction-diffusion systems with diagonal diffusion matrices, where Turing-wave instabilities are those related to spatiotemporal patterns. In this talk, I am going to talk about the generalization of these conditions to arbitrary reaction-diffusion equations in the presence of linear cross-diffusion, and I will also highlight some natural generalizations of the theory to general reaction-diffusion equations in the presence of nonlinear (cross-) diffusion. Furthermore, I am going to show examples of spatiotemporal patterns in cross-diffusion systems that cannot have Turing or Turing-wave instabilities without cross-diffusion, together with the ideas used to assemble these examples.

How Classical is Fuzzy Dark Matter

Mx Alex Gough

Newcastle University

Abstract:

Fuzzy dark matter treats the dark matter component of the universe as a single wavefunction with a macroscopic de Broglie wavelength. Such systems arise from many ultralight dark matter candidates, and feature rich and universal interference phenomena. Full numerical simulations of such systems are currently limited to small box sizes, leaving open questions about the impact of such interference on the cosmic web. Attempts to go to larger scales include "classical fuzzy dark matter" which runs standard N-body particle simulations on modified initial conditions which mimic the suppression of small scale structures present in fuzzy dark matter. Such an approach is free of other interference features which would occur in the full wave dynamics. I will discuss an analytic, perturbative, model of wavelike dark matter, which allows us to keep these interference effects in the final matter field. In a toy model we demonstrate how interference in wave systems arises as a "weaving" of classical particle trajectories, linking classical multi-streaming to interference patterns. In perturbative simulations of the cosmic web, we examine how changing initial conditions and changing interference dynamics separately impact cosmological statistics. This allows us to investigate how classical these wavelike systems are on scales of the cosmic web.

Pattern formation and stability for a kinetic model of ants

Mr Oscar de Wit

University of Cambridge

Abstract:

We present an interacting particle system to model the behaviour of collectives of ants. The particles are modelled as Active Brownian Particles interacting only via chemical pheromones. The particles also have an antenna by which they sense the pheromones. The particles form typical Keller-Segel collapse clusters or, depending on the length of the antenna, travelling clusters. We study the formal mean field limit PDE model to substantiate these particle behaviours. We show analytical and numerical results for the PDE. We begin by demonstrating analytical well-posedness and uniform boundedness globally in time. Using a convergent scheme we are also able to obtain the linear instability curve in the parameter space. In the linearly unstable regime finite volume simulations demonstrate that there is pattern formation reflecting either the Keller-Segel collapse or lane formation, depending on the length of the antenna and the interaction strength. The patterning is also explained by the shape of the growing eigenfunctions associated to the linear theory. We also show there is a region of bistability where Keller-Segel collapse and lane formation compete.

Data-driven equation discovery for liquid film flows thick and thin

Mr Sebastian Dooley

University of Warwick

Abstract:

Partial differential equation (PDE) discovery is an exciting alternative to the standard first principles-based methodologies regularly used in mathematical modelling, particularly in regimes outside the reach of traditional approaches. This talk focuses on the application of PDE discovery methods for liquid film flows with the aid of direct numerical simulation data. To begin with, we focus our attention to the potential recovery of established thin film equations, outlining important derivation aspects to build analytical understanding into the data-driven process. We then gently steer the developed framework into new regimes of interest, such as thick liquid film flows, in which classical physical understanding is more sparse.

Triple Deck Theory by a Green's integral equation

Dr Edmund Chadwick

University of Salford

Abstract:

Consider steady two-dimensional incompressible flow past a finite flat plate for large Reynolds number. At the trailing edge a triple deck occurs. Classically, this is representation is obtained by using matched asymptotic expansions that rescale lengths in the axial and transverse directions to find the interactions between the decks. Also required in the matching are the Blasius solution for flow past a semi-infinite flat plate and the near-field Goldstein wake solution which match asymptotically upstream and downstream respectively of the triple deck region. A reoccurring term in the interactions is found which is the lateral flow displacement function.

Recently a new theory for the Green's integral representation for the Navier-Stokes velocity has been given in terms of Green's functions called NSlets. The question asked here is can this representation can be used to reproduce the triple decks, where for each deck a different approximation to the NSlet is used? It is shown that it can, providing a benchmark for test giving confidence in the new theory as well as giving insight into our understanding of the triple deck. In particular, the negative derivative of the lateral flow displacement function is identified to be proportional to the strength function of the NSlets in the Green's integral representation.

Topological Data Analysis of Monopoles in U(1) Lattice Gauge Theory

Mr Xavier Crean

Swansea University

Abstract:

It has been widely argued that non-trivial topological features of the Yang-Mills vacuum are responsible for colour confinement. However, both analytical and numerical progress have been limited by the lack of understanding of the nature of relevant topological excitations in the full quantum description of the model. Recently, Topological Data Analysis (TDA) has emerged as a widely applicable methodology in data science that enables us to extract topological features from data. We explain how TDA paired with machine learning may be used to quantitatively analyse the deconfinement phase transition in 4d compact U(1) lattice gauge theory by constructing observables built from topological invariants of monopole current networks.

Melting of wall-mounted ice in shear flow in two dimensions

Miss Ellen Jolley

University of Oxford

Abstract:

Ice accretion on aircraft surfaces deforms the aerodynamic shape and degrades performance, in worst cases resulting in serious accidents. Mathematical modelling of this process could enable improved predictive codes to be developed which can more accurately predict the safety of air vehicles. The melting and freezing of ice on a solid surface is therefore of significant industrial interest. In this work, we model an ice hump attached to a solid surface within a boundary layer flow. We present a mathematical model to describe the nonlinear interplay between the ice phase change, surrounding flow, and heat transfer in the fluid, assuming uniform shear flow and constant heat transfer in the undisturbed fluid near the wall. The flow and heat equations are assumed to be quasi-steady due to the slow evolution of the ice hump shape. Considering a small ice hump and taking small perturbations from the flow profile produces a linear problem which can be solved in Fourier space using Airy functions, allowing the ice shape evolution to be investigated analytically and numerically for both warm and cold incoming flow. A critical value for heat transfer is found, below which freezing take place and above which melting takes place. In the melting case, the ice hump disappears in finite time, whereas in the case of freezing the asymptotic behaviour of the ice hump edge points at large times can be found analytically.

Current trends in the COVID-19 pandemic dynamics

Professor Igor Nesteruk

SBIDER (Systems Biology & Infectious Disease Epidemiology Research) Centre at University of Warwick, UK

Abstract:

The interest in the COVID-19 pandemic declines despite still high mortality (e.g., in 2023 in the UK it was 4 times higher than the global value caused by seasonal flue [1]). The accumulated numbers of cases CC, deaths DC, tests TC, fully vaccinated persons VC, and boosters BC per capita listed in John Hopkins University datasets, calculated daily characteristics DCC, DDC and DTC , and case fatality risks CFR=DC/CC values were used to compare the pandemic characteristics in 2020-2024 [1-8] and to find linear and non-linear correlations [1, 3-6]. It was shown that the population age A is a pivot factor in visible (registered) part of the COVID-19 pandemic dynamics [1, 5]. Much younger and less vaccinated Africa has registered much less numbers of cases and death per capita due to many unregistered asymptomatic patients [1, 5, 6]. With decreasing of testing level DTC the case fatality risk can increase drastically [1]. CC, DC, DCC, and DDC values increase with increasing VC and BC, calling into question the effectiveness of vaccinations [1, 3, 6]. The influence of the Ukrainian refuges on the COVID-19 pandemic dynamics [7] and endemic characteristics of SARS-CoV-2 disease [8] are discussed.

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6. 10.13140/RG.2.2.11465.52327
7. https://doi.org/10.20944/preprints202401.0733.v1
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Potential features of electroencephalogram extracted by information geometry and fractal dimension

Mr Heng Jie Choong

Coventry University

Abstract:

Past studies have shown that the brain activity at resting state, which is represented via whole-brain functional connectivity (FC), would deviate to different states. However, this distinct change of the states is fully used as the features for the diagnostic purpose. In this study, I have utilised this changing of FC states of electroencephalogram (EEG) signals as the features for Support Vector Machine (SVM). In details, the FC matrices at each state of the EEG are first reduced to be represented by the leading eigenvectors to capture the dominant FC pattern. These eigenvectors are ensemble to form the probability distribution via the distance, specifically cosine distance, between the eigenvectors which was later used to quantify the change of the probability distribution. This change of distribution is known as the information rate which is used as the features for the SVM. Along with information rate, the fractal dimension of the signal states are quantified as the features too. Using all of these features at different frequency bands of the EEG signals, the SVM has cross-validated by 20 folds and managed to classify healthy and non-healthy patients (Alzheimer's and Frontotemporal patients) with the accuracy of 67+/-2%.

Three dimensional melting of wall mounted ice in shear flow

Ms Thuy Duong Dang

University College London

Abstract:

Ice formation on various surfaces have been known to pose efficiency issues and safety hazards, particularly in aeroplanes where ice growth on wings, deforming the overall wing shape, can pose a significant risk to the aircraft. Ice formation inside domestic or commercial pipes, can also presenting plumbing issues in buildings. The majority of icing literature focuses on equivalent two-dimensional models however there is clearly scope for improvement as more complex three-dimensional models present a much more realistic scenario. We present a novel approach to modelling a three-dimensional ice lump of finite extent melting on a flat wall, with the oncoming fluid surrounding the ice, being warmer than the ice. The wall temperature is assumed to be the same as the near wall water, except for the area directly underneath the ice lump, which is the same temperature as the ice. The flow behaviour is studied both analytically and computationally and in-flow predictions of the ice shape are presented as well as the induced pressure gradient caused by the flow.

Surface-tension-driven buckling of a viscous disc

Mr Nicholas Ryan

University of Oxford

Abstract:

In the manufacture of thin glass sheets, e.g. for smartphone and tablet screens, buckling can cause ripples to become set into the final product, making it unusable. In this talk we present a simplified model problem which attempts to capture how this spontaneous buckling – driven by local compressive stress – can occur. We model a thin disc of fluid retracting under surface tension and consider how initial sinuous and varicose perturbations affect the shape evolution. Furthermore, we examine which modes are dominant and how this depends on the stresses in the disc.

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Comparisons of internal solitons of the MCC model against extended KdV equation

Mr Nerijus Sidorovas

Loughborough University

Abstract:

For a two-layer flow, plane waves are considered within the scope of the Miyata-Choi-Camassa (MCC) model in the rigid-lid approximation. We derive the extended Korteweg-de Vries (eKdV) equation for the long internal waves and use a near-identity transformation to reduce it to the Gardner equation. In a case study, we numerically compare solitons of the eKdV equation, including the table-top solitons, against their evolution in the MCC model.

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Predicting internal collapse of biological filament bundles

Dr Christopher Prior

Durham University

Abstract:

I will discuss a flexible and and adaptable model of biological filament bundles which we used to predict the mode of internal collapse of the bundle. This model can capture physically observed deformations such as global buckling, pinching and internal collapse. We explored the transitions between these deformation modes numerically, using an energy minimization approach, highlighting how supported environments, or stiff outer sheath structures, favour internal structural collapse over global deformation. I will present novel analytic buckling criterion for the internal collapse of the system, a mode of structural collapse pertinent in many biological filament bundles such as the optic nerve bundle and microtubule bundles involved in cell abscission. Finally I will discuss recent extensions of this model and how it could be tailored to specific applications in order to make use of its predictive efficacy.

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VisualPDE: Playing with Patterns via Web Browsers

Dr Andrew Krause

Durham University

Abstract:

At the BAMC 2023 we unveiled VisualPDE.com for the interactive solution of a large class of partial differential equations using lightning-fast interactive solvers inside of web browsers. Since then we have greatly expanded the capabilities of the website, and begun using it intensively for both teaching and research.

In this talk I will comment on aspects of modelling pattern formation, and other biological and physical processes using VisualPDE.com. I will showcase the valuable insights one can gain by rapidly prototyping different models, and sharing them with collaborators (even those far outside of mathematics). I hope to encourage others to engage with this kind of technology to 'play' more with the science we do, both for the improvement of our own research and teaching, as well as for public understanding.

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Curvature effects on two-phase flow for optimal underground hydrogen storage

Mr Peter Castellucci

University of Manchester

Abstract:

In the effort to slow the rapidly deteriorating climate crisis through the use of cleaner energy sources, underground hydrogen storage presents a promising way to store surplus energy from renewable sources. Motivated by CO_2 sequestration, the flow of an ambient fluid in a porous medium displaced by an injected fluid of different density and viscosity has been intensively studied. The distinct challenges posed by hydrogen's order of magnitude lower viscosity at high pressures and its cyclical storage demand innovative approaches. The dynamics of these flows are determined by spreading due to buoyancy gradients and the pressure gradient from injection. Using a long-wave approximation, we derive the nonlinear advection-diffusion equation governing the evolution of the hydrogen-brine interface in the curved geometry of depleted oil/natural gas reservoirs. Local mass conservation is used to determine the interface behaviour at the free boundaries. We demonstrate how using a distributed source instead of a point source can avoid the singular boundary condition at the origin which arises in axisymmetric models. The flow dynamics depends on two parameters, the mobility ratio M (ratio of viscosities), and λ, the relative effect of buoyancy to viscous forces. Our results show that the small mobility ratio of a hydrogen-brine system results in significant lateral spreading, which may lead to losses of hydrogen if the current reaches beyond the spill point. The spreading can be diminished by lowering the injection rate and hence increasing λ so the flow enters a buoyancy dominated regime causing the interface to lower horizontally.

Multiple-scale analysis of a premixed flame in confinement

Mr James Harris

University of Oxford

Abstract:

Flames are subsonic combustion waves, often with Mach numbers significantly smaller than unity. As such, flames generate acoustic waves in the gas surrounding them. The acoustic waves travel at the speed of sound and, in an enclosed volume, the repeated reflection of the acoustic waves between the boundaries and the comparatively slow flame cause the temperature, and hence the speed of sound, to rise in the unburnt gas. To account for the changing sound speed, we use an adaption of the method of multiple scales in the limit of small Mach number to analyse a one-dimensional mathematical model for a flame in a confined volume. We find that the leading-order perturbation to the pressure can be written as a series of modes whose spatial structure, amplitude and frequency are all slowly varying.

Spectral Measures for Graph Classification Problems

Dr Ka Man (Ambrose) Yim

Cardiff University

Abstract:

The graph Laplacian is a fundamental tool in network science and graph signal processing. In particular, its eigenvalues and eigenvectors are known to encode fundamental structural properties of the underlying graph. We propose a novel way of encoding both eigenvalues and eigenvector information in a set of `spectral measures' that describes the local geometry of the graph at each node. By transforming the set of eigenvectors into spectral measures, we can define a notion of similarity between vertices, even if they come from graphs with different eigenvector dimensions. We show how spectral measures naturally encode elementary properties of the graph in its moments, and generalise the heat kernel signature and wavelet signature used for graph signal processing. Using spectral measures as features for graph classification problems, we are able to achieve comparable performance with state of the art graph neural networks on benchmark graph classification tasks.

Controlling stratification in drying films with electrolyte-driven diffusiophoresis

Dr Clare Rees-Zimmerman

University of Oxford

Abstract:

Stratification in drying films – how a mixture of differently-sized particles arranges itself upon drying – is examined. Being able to control this would allow the design of coating formulations which self-assemble during drying to give a desired structure. Potential applications are across a range of industries, from a self-layering car paint, to a biocidal coating in which the biocide stratifies to the top surface, where it is required.

Diffusiophoresis is the migration of particles along a concentration gradient of a different solute species. Recent experimental work has highlighted that electrolyte-driven diffusiophoresis is likely to be the most significant diffusiophoretic motion in a mixture of silica nanoparticles and relatively large latex particles, which are commonly used in coatings. In this present work, this diffusiophoretic effect, powered by gradients in the nanoparticles and their stabilising cations, is modelled in drying films.

A continuum hydrodynamic model is derived, and the resulting partial differential equations solved numerically. An asymptotic solution is found for high evaporation rate. It is found that the final film structure is governed by the relative magnitudes of the diffusive and diffusiophoretic terms. Two methods are discovered to control the resulting stratification: (i) setting the surface charge on the particles, and (ii) setting the background salt concentration. Either of these can be used to select either small- or large-on-top stratification or a homogenous film. The diffusiophoretic term promotes small-on-top stratification, and so may account for experimental observations of accumulated small particles at the top surface of dried films.

Kinetic modelling explains heterogeneity in red-blood-cell properties in sickle-cell disease

Miss Claudia Alicia De Sousa Miranda Perez

University College London

Abstract:

Sickle-cell disease (SCD) is a genetic blood disorder with symptoms induced by the polymerisation of haemoglobin inside red blood cells (RBCs) in reduced oxygen tension. Despite decades of intensive investigation of SCD aggregation mechanisms, cell-resolved measurements of haemoglobin polymerisation in whole cell populations are only recently starting to be made. These experiments reveal that SCD saturation distributions at intermediate oxygen tensions are bimodal, suggesting the existence of two subpopulations of RBCs in SCD patients — one with a significant amount of polymer, and the other with fewer polymers present. New theoretical frameworks are needed to explain the mechanisms behind this heterogeneity. Here, we present approximate analytical expressions for dynamic polymer content in an RBC population in constant and fluctuating oxygen tensions. Our results will help in the search for universal patient biomarkers that can be clearly related to SCD outcomes, aiding the effective treatment of the disease.

The coastal front propagation from a river outflow governed by dispersive potential-vorticity dynamics

Mr Michael Nguyen

University College London

Abstract:

This report studies into the propagation of coastal currents from a river outflow using a 1-1/2 layer, quasi-geostrophic (QG) model following Johnson, Southwick and Mcdonald 2017 (JSM17). Briefly, the model allows fluid from a river outflow to be propagated via the Coriolis effect driven by a Kelvin wave (KW) and the image-effect driven by a coast due to the potential vorticity (PV) of the expelled river fluid. We analyse the long-wave dispersive or first-order asymptotic equation governing the evolution of the boundary of the coastal current, also known as a (coastal) PV front, extending from the leading-order hydraulic analysis from JSM17 which did not explain key structures or predictions of some current-widths along the coast predicted by the full QG equations. We argue that adding the first-order (dispersive) term is sufficient to mathematically explain the structures present as well as the sizes of current widths in the entire full QG problem even in the narrow width limit where the long-wave approximation is expected to break down. We also derive an upper and lower bound on the steady current widths, measured as the current distance from the coast to the fluid propagation, and consolidate how we can build upon the hydraulic solutions to link these analyses together.

The initiation, invasion and blockade of ischaemic Alzheimer's disease

Mr Andrew Ahern

University of Oxford

Abstract:

The role of vascular degeneration in Alzheimer's disease (AD) has received increased attention in recent years. The key disease agent of AD, amyloid beta, has been found to effect an increase in the hydraulic resistance of the brain's microvasculature via multiple mechanisms. The resulting ischaemia, conversely, is known to effect an increase in amyloid beta production. It has been argued that together these pathological mechanisms underlie a vicious cycle that culminates in the build-up of amyloid beta and downstream neurodegeneration.

In this talk, we aim to formalise such hypotheses in a multiscale mathematical model with a view to investigating their consequences for disease initiation, its invasion of neighbouring brain regions, and the possibility of containing the disease with pharmaceutical treatment. By treating amyloid beta as a prion-like protein, capable of spreading much like an infectious disease through the human connectome network, and the amyloid-induced rarefaction of cortical capillary networks as a dynamic percolation process whose effect on the flow rate we can quantify, we show that bistable dynamics emerge, giving rise to a variety of rich spatiotemporal behaviours. In particular, the theoretical possibility arises of blockading the disease's spatial advance by forming a boundary region of tissue that is treated with a drug. The requirements of such a barrier will be discussed.

Visualizing Droplet Friction on Liquid-Infused Surfaces

Dr Abhinav Naga

Durham University

Abstract:

Liquid-infused surfaces (LIS) are a class of lubricated surfaces that have an exceptional ability to repel a wide range of liquid droplets. Compared to the 'lotus effect' wetting dynamics on traditional liquid-repellent surfaces, droplets on LIS exhibit unique wetting dynamics due to the formation of a lubricant meniscus (called the wetting ridge) around the droplet. Despite the importance of the wetting ridge in governing droplet dynamics, it has been challenging to directly visualize and validate the mechanism by which it dissipates energy and generates friction during motion.

Here, we use lattice Boltzmann simulations to visualize energy dissipation inside droplets and their wetting ridges. Focussing on the common limit where the lubricant is more viscous than the drop, we find that energy is predominantly dissipated in the lower extremities of the wetting ridge directly in front and at the rear of the drop. Significantly less dissipation occurs at the lateral sides of the drop and in the lubricant film underneath the drop. We further perform experiments using confocal microscopy to image the shape of the wetting ridge during motion and compare these to our simulations. We conclude that although the dissipation heatmaps for 2D and 3D simulations are remarkably similar, only 3D simulations reproduce the shape and size wetting ridge accurately. These insights will inform future studies that model drop friction and lubricant transport on lubricated surfaces.

Scattering of an Ostrovsky wave packet in a coupled waveguide

Mr Jagdeep Tamber

Nottingham Trent University

Abstract:

In this talk we will explore the scattering of an Ostrovsky wave packet, generated from a solitary wave, in a two layered waveguide with a delamination in the centre and soft (imperfect) bonding either side. The materials in the layers are distinct, leading to a system of coupled Boussinesq equations. Direct numerical modelling is difficult, so we use a semi-analytical approach consisting of several matched asymptotic multiple-scales expansions, which leads to Ostrovsky equations in soft bonded regions and Korteweg-de Vries equations in the delaminated region. The semi-analytical approach and direct numerical simulations are in good agreement. The dispersion relation is used to determine differences between the case without delamination and when a delamination is present in the structure. We identify a phase shift in the final bonded region, compared to the case without delamination, for various delamination lengths. This can be used as a tool to control the integrity of layered structure, as we will show that we can determine the size and position of a delamination, without prior knowledge of its placement in the structure, and we will present examples to justify the results.

Interpolation between the Navier-Stokes and the solenoidal Burgers equations

Professor Koji Ohkitani

RIMS, Kyoto University

Abstract:

The multi-dimensional Burgers equations are integrable in that they can be reduced to a heat equation under the assumption of potential flows via the so-called Cole-Hopf linearisation. On the other hand, it is believed that the Navier-Stokes equations are not.

In two spatial dimensions, by rotating the velocity gradient by 90 degrees we can obtain an equation which is equivalent to the Burgers equation. Rotating the velocity gradient continuously we introduce a generalised system of incompressible fluid equations that allows us to compare an integrable system with non-integrable ones. Based on direct numerical experiments of the generalised system, we find that the Burgers equation is the most singular in terms of enstrophy.

We then discuss preliminary results on a similar extension in three spatial dimensions using vector potentials. We compare numerically the solenoidal Burgers and the Navier-Stokes equations with a regularity criterion integral involving enstrophy.

Why care about inverse problems when your focus is on modelling?

Dr Art Gower

University of Sheffield

Abstract:

This will be a light hearted talk on the basics of inverse problems aimed at an audience who develop models of linear waves (acoustics/elastic/electromagnetic), but do not currently work on inverse problems. I will illustrate the basics of inverse problems in the context of de-blurring an image [1], and cover classical regularisation and motivate statistical inverse problems. There is code available [2] written in Julia to run the examples of this talk.

References:
[1] Kaipio, Jari, and Erkki Somersalo. Statistical and computational inverse problems. Vol. 160. Springer Science & Business Media, 2006.

[2] https://github.com/arturgower/InverseProblemsExample.jl

Colloidal deposits from evaporating sessile droplets

Mr Nathan Coombs

University of Warwick

Abstract:

The coffee ring effect (CRE) refers to the accumulation of solute particles near the contact line of an evaporating sessile droplet and arises due to evaporation-induced capillary flow. Suppression of the CRE is desirable in many industrial applications which utilize colloidal deposition from an evaporating liquid, notably inkjet printing. It is therefore important that the influence of experimentally accessible physical parameters (ambient temperature, humidity, particle size/shape etc.) on the deposit morphology are well understood.

Of critical importance in CRE modelling is the inclusion of particle "jamming": when solute reaches a threshold volume fraction (approximately 64% for mono-disperse spherical particles), a transition towards a porous solid is observed. Jammed particles have a semi-crystalline structure and can exhibit both ordered and disordered phases depending on the local advection speed. Since jammed solute is incompressible, it also influences the shape of the drop's surface, ultimately leading to a reversal in surface curvature and meniscus touchdown at the late stages of evaporation.

Existing CRE models that include jamming are limited in scope to pre-touchdown dynamics and so are not able to describe the drying process in full. In this talk I will introduce a modelling framework that remedies this issue. Though much of the focus will be on axisymmetric drops, the model can be easily generalised to arbitrary drop shapes, allowing us to explore the influence of contact line curvature on the local CRE intensity.

Time permitting, I will also look at the role of particle assembly at the drop surface and how it can be exploited to attenuate the CRE.

An enthalpy model for modelling of ice crystal icing in engines

Mr Timothy Peters

University of Bath

Abstract:

In the last decade, it has been noted that ice crystal icing (ICI) is responsible for more than a hundred observed instances of power loss and engine damage. In this scenario, ice crystals present at high altitudes enter the hot engine core. Despite the high temperatures in the core, the crystals can accrete on the interior surfaces in a partially melted state, and then shed outwards, damaging key components, and causing power loss. Existing knowledge on the mechanism of ice crystal build-up and shedding is still limited, although some models have been recently developed.
In this talk, we discuss the derivation and study of an enthalpy model of ice and water accretion over a heated substrate, subject to a combination of ice/water heat flux conditions. We compare the results with a previously established multilayer thin film model in the icing crystal icing literature.

Electrified pendent liquid bridges

Ms Agnes Bokanyi-Toth

Loughborough University

Abstract:

We study pendent liquid bridges between two horizontally aligned cylindrical rods using experiments, direct numerical simulations and reduced-order model equations obtained by minimising an appropriate Rayleighian according to Onsager's variational principle. Additionally, we analyse the influence on the dynamics of perfect dielectric liquid bridges of the electric field resulting from an imposed potential difference between cylindrical electrodes. Our findings reveal that the electric field pulls the liquid upwards and flattens the liquid-air interfaces. Also, we find that the maximum trapping capacity (i.e. the amount of fluid that can be captured between the cylinders) is increased in the presence of the electric field. The experiments are performed using silicone, castor, and mineral oil droplets and three types of cylindrical electrodes: titanium, copper, and stainless-steel rods. We observe that the experimental results are well described by the model equations for a wide range of parameter values. Since liquid bridges are a reasonably good model of Plateau border cross-sections in a liquid foam, by understanding the dynamics in such geometry, we gain better insight into electrohydrodynamic phenomena in foams. Thus, this project is contributing to tackling key problems related to the control and stability of liquid foams.

Including dynamic organism and environmental heterogeneity in collective behaviour: looking at locusts

Dr Fillipe Georgiou

University of Bath

Abstract:

Collective behaviour occurs at all levels of the natural world, from cells joining together to form complex structures, to locusts interacting to form large and destructive plagues. These complex behaviours arise from simple individual and environmental interactions, and thus lend themselves well to mathematical modelling. One simplifying assumption, that of relative homogeneity of organisms, is often applied to keep the mathematics tractable.However, heterogeneity arising due to the internal state of individuals has an impact on these interactions and thus plays a role in group structure and dynamics.

Through the lens of locust foraging, I introduce a continuum model that accounts for this heterogeneity in the form of a state space that maps this internal state to movement characteristics. Then using the model we explore the effect of food, hunger, and gregarisation on locust group formation and structure. Finding that the most gregarious and satiated locusts tend to be located towards the centre of locust groups (Conversely, hunger drives locusts towards the edges of the group). Finally, we find that locust group dispersal may be driven in part by hunger.

These results lend themselves to better outbreak prediction both in terms of the initial aggregation formation as well as the eventual dispersal. In addition, the technique itself is general enough that could potentially be used to explore a variety of different scales from microscopic to macroscopic.

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Dipolar geophysical vortices

Professor Edward Johnson

University College London

Abstract:

Recently analysis of satellite altimetry and Argo float data led Ni et al. to argue that mesoscale dipoles are widespread features of the global ocean having a relatively uniform three-structure that can lead to strong vertical exchanges. We show that almost all the features of the composite dipole they construct can be derived from a model for multipoles in the surface quasi-geostrophic equations for which we present a straightforward novel solution in terms of an explicit linear algebraic eigenvalue problem, allowing simple evaluation of the higher radial modes that appear to be present in the observations and suggesting that mass conservation may explain the observed frontogenetic velocities.

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Modelling epileptiform discharges as frequency synchronization of Kuramoto oscillators

Dr Yingjing Feng

University of Birmingham

Abstract:

The epileptiform discharge (ED) is an important clinical biomarker on the electroencephalograms (EEGs) for diagnosing epilepsy. Epilepsy is a serious neurological condition characterised by seizures. Seizures are increasingly understood as emerging due to synchronization across large-scale brain networks. To investigate the genesis of generalized EDs, we modelled their occurrence as frequency synchronization across a modular network of Kuramoto oscillators. We computed the basins of attraction of the equilibria to investigate their stability under perturbations. Informing the network structure from a dataset of 24-hour EEGs from 101 people with epilepsy, we found that the model explains phase cohesiveness patterns of generalized EDs both within and between subjects. Future works lie in explaining the temporal distribution of generalized EDs during the wakefulness and non-REM sleep states, and designing new routes for better seizure control.

Mean first passage time and its application in ocular drug development

Mrs Patricia Lamirande

University of Oxford

Abstract:

Wet age-related macular degeneration is a progressive disease that leads to severe visual impairment. Standard of care treatment involves drug injections into the eye, determining the yet unmet medical need of reducing injection frequency. This motivates modelling efforts to understand and help select prospective drugs with longer retention times. To this end, we developed a mean first passage time (MFPT) modelling framework, to investigate the scaling relationships of ocular pharmacokinetics in humans and animal species and to inform drug development.

The MFPT describes how long it takes, on average, for a random walker to reach a given target, and is a valuable method to quantify the efficacy of diffusion transport. In this work, we derived a partial differential equation system that describes the MFPT of a particle in a 3D finite domain, bounded by reflective and semi-permeable conditions, modelling the diffusion of a drug injected into the eye. We applied our model to quantify the influence of anatomical and physiological parameters of the eye on the kinetics of protein therapeutics, to better relate retention times between different species. We also investigated the impact of the injection location and of the variability in human eyes on drug elimination.

Homogenised properties of viscoelastic composites: an asymptotic homogenisation perspective

Mr Alejandro Roque-Piedra

University of Glasgow

Abstract:

This research focuses on studying the mechanical properties of viscoelastic composites. The analysis utilises the asymptotic homogenisation technique [1-3] which is employed to separate the equilibrium equation into cell and homogenised problems for viscoelastic composites. The general theory is specialised in the case of a Saint-Venant-type strain energy density, with the second Piola-Kirchhoff stress tensor also featuring a viscous contribution. Within this setting, we frame the general theory in the case of infinitesimal displacements to use the correspondence principle which results from applying the Laplace transform. This choice is also advantageous to avoid the numerical complications arising in a finite theory. Taking inspiration from [4], we specialise in the analysis of fibre-reinforced composites and provide semi-analytical solutions to the cell problems and closed expressions for the effective coefficients. Constitutive models for the memory terms are chosen to simulate benchmark problems, and our results are shown to converge rapidly and align well with existing literature data. Future developments aim to explore diverse microstructural geometric arrangements in nonlinear viscoelastic composites and extend the analysis to biological scenarios, including fibrous tissues such as muscles and connective tissue. It is expected that further research in this area will lead to new research questions in materials science and biomathematics.

References:
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Boundary layer recursion parameter selection for the one-way Navier-Stokes Equations

Mr Elliot James Badcock

Imperial College London

Abstract:

In recent years, the one-way Navier-Stokes (OWNS) methodology has been introduced as a spatial-marching technique used to investigate the stability of flows with a slowly varying direction, alongside the parabolised stability equations (PSE).

The PSE equations require disturbances propagating in the boundary layer to possess a modal ansatz in the streamwise direction, alongside a closure relationship that allows a partition and split of the rapid variation of the disturbance into an exponential term. The inherent ellipiticity of the system isn't fully removed, but instead manifests itself as numerical instabilities in the spatial march if the step size is sufficiently small.

Although OWNS has a greater CPU cost, unlike PSE the marching direction of OWNS has no minimum step size restriction, and as such OWNS can resolve all disturbance length scales. Despite this obvious improvement, the most crucial step in the OWNS methodology, that of selection of so-called recursion parameters that parabolise the system, is a very challenging task to generalise for all flows.

During the presentation we will give an overview of the OWNS methodology, critically assess its merits over the conventional PSE, and discuss recent improvements to a recursion selection algorithm which correctly determines the recursion parameters for a wide variety of boundary layer flows – including a flow that features a laminar separation bubble.

Volcanic fissure localisation: Thermoviscous fingering in non-uniform geometries

Dr Jesse Taylor-West

University of Bristol

Abstract:

When volcanic lava erupts through a fissure, the system often evolves from a uniform sheet flow to a system of distinct localised vents. One proposed mechanism for this behaviour is via a thermoviscous fingering instability in which hot, low-viscosity lava forms finger-like preferential pathways through cooled higher-viscosity lava, analogously to the classical Saffman-Taylor instability. Previous work has considered the flow of hot fluid driven by an imposed pressure drop through a planar fissure with walls held at a constant, colder temperature. These results have demonstrated non-uniqueness of steady state solutions, and explored the instability of solutions to non-planar perturbations. In this talk I will extend these results to non-uniform fissure width, as is often the case for real-world volcanic fissures. I will show how this modifies the bifurcation diagram of the system and the conditions for instability, and discuss the implications for the localisation behaviour of volcanic fissure eruptions.

Mode analysis in quasiperiodic phononic crystal structures waves

Dr Marc Martí Sabaté

Imperial College London

Abstract:

Multiple scattering theory facilitates the computation of states and scattering fields on a diverse variety of wave systems. Through rigorous analysis, this theory yields insights into wave phenomena such as diffraction, reflection, and transmission, thereby contributing to a comprehensive understanding of the complex dynamics of classical systems. Our investigation specifically delves into flexural waves within thin elastic plates, with point-like mass spring resonators serving as scatterers attached the plate's top surface. Finite clusters of scatterers will be analysed without imposing symmetries or periodicity in the system. Despite the seemingly straightforward nature of this model, it proves capable of unveiling and predicting many behaviours related to the cutting-edge topics on metamaterials, including phononic crystals, quasicrystals, topological insulators, rainbow trapping effects or spatio-temporal modulation.

This presentation addresses two distinct structures: one-dimensional quasi-periodic arrays of scatterers and twisted bilayers. In both cases, specific parameters enable the transition from commensurate to incommensurate structures. The emergence of modes near the commensurate phases of the structure, as well as the existence of edge modes due to the finiteness of the cluster are studied, emphasizing their high quality factor relative to the structure's propagating modes. Results underscore the remarkable tunability of quasi-periodic structures and their potential for the design of wave-trapping devices.

Oscillatory reconnection at null points in the solar atmosphere

Dr Gert Botha

Northumbria University

Abstract:

Null points are places in a magnetic field where the size of the field goes to zero. In the solar atmosphere they occur due to the complex magnetic configuration generated by the sun. One important physical process associated with null points is oscillatory reconnection. A short description of this time-dependent reconnection process in plasmas will be given. From numerical simulations we gain insight into the properties and dependencies of this process. We shall be looking at these numerical results and show how they help us to diagnose the solar coronal plasma.

On the transition to dripping of an inverted liquid film

Dr Dmitri Tseluiko

Loughborough University

Abstract:

The gravity-driven flow of a liquid film under an inclined plate is investigated at zero Reynolds number. Travelling-wave solutions are analysed assuming either a fixed fluid volume or a fixed flow rate for two thin-film models with either linearised or full curvature (the LCM and FCM, respectively) and the full equations of Stokes flow. Of particular interest is the breakdown of travelling-wave solutions as the plate inclination angle is increased, which is associated with the onset of dripping and which is analysed by asymptotic analysis and by constructing bifurcation diagrams for a wide range of parameters. It is found that the thin-film models either provide an accurate prediction for dripping onset or else supply an upper bound on the critical inclination angle [1]. The predictions from the asymptotic analysis and bifurcation diagrams are corroborated by direct numerical simulations for the Navier-Stokes equations using the open-source volume-of-fluid Gerris software.

References:
[1] M.G. Blyth, T.-S. Lin, D. Tseluiko (2023) On the transition to dripping of an inverted liquid film, J. Fluid Mech. 958, A46

Modelling structural vibrations using phase space: Extending ray dynamics

Mr Rory Colette

University of Nottingham

Abstract:

Ray tracing techniques are a key technique in the modelling of structural vibrations, however a fundamental flaw in their use is their ignorance of significant aspects of wave behaviour. Interference, diffraction, and other wave effects are inherently ignored by modelling the transport of vibrational energy through the lens of ray dynamics, thus we seek a new approach by turning to phase space. We are developing an alternative modelling approach, which exploits Wigner and Husimi functions, to describe the phase space of structural vibration problems. We replicate results seen from taking Eikonal approximations and the resulting ray picture, and display that our phase-space method observes the same features. We present the basis for our approach and a quasi-1D curved shell example, to display that the resulting phase space manifests the effects of the structure of the system. The purpose of this approach is we can consider the phase space of a randomly driven system, and retain the wave effects while evolving the system in the phase space. The Wigner function's close relationship with the autocorrelation function will then allow for a more accurate prediction of acoustic radiation, which is the ultimate aim of the project.

Flame propagation and instabilities in a plane Couette flow

Dr Joel Daou

University of Manchester

Abstract:

In the presence of shear-enhanced diffusion (Taylor dispersion), flame propagation is effectively anisotropic. The study addresses the influence of the direction of a strong plane Couette flow on the propagation of premixed flames and their instabilities. The presentation will mainly focus on the thermal-diffusive (Turing) flame instabilities within a constant density model, but will also briefly present recent analytical results on the Darrieus-Landau instability associated with density variation.

For the thermal-diffusive instabilities, the study compares analytically derived stability results with simulations involving the Lewis number Le, the Peclet number Pe, and the angle between the direction of propagation and the flow direction. Cellular instabilities expected when $Le < 1$ are now found to occur due to Taylor dispersion in $Le > 1$ mixtures, provided the angle exceeds 75 degrees. Particular attention is devoted to the cellular long-wave instability encountered, which is found to be described by a modified Kuramoto-Sivashinsky equation. The equation involves the three aforementioned parameters and includes a dispersion term (a third-order spatial derivative) as well as a drift term (first-order derivative).

For the Darrieus-Landau instability, a dispersion relation is derived analytically incorporating the effect of shear-enhanced diffusion and its implications are discussed.

The emergence of non-standard growth laws from systematically upscaling heterogeneous organoids

Dr Meredith Ellis

University of Birmingham

Abstract:

Organoids are three–dimensional multicellular tissue constructs used in applications such as drug testing and personalised medicine. Organoid culture methods, e.g. using bioreactor systems, typically incorporate a hydrogel in which millions of growing organoids are embedded. Mathematical models of this process (transport of metabolites within the bioreactor) cannot account for each individual organoid over the entire bioreactor due to the prohibitive computation expense required, and instead use effective terms.

In this talk, we will present a systematic derivation of the appropriate effective terms to include when microscale variation is important, as in the case of organoid growing within a hydrogel, inducing a time-dependent underlying microstructure. We upscale the microscale problem using homogenisation via the method of multiple scales, focusing on the distinguished limit where the microscale heterogeneity of metabolite concentrations appears at leading order. We find that the classic volumetric scaling for effective uptake and production argued from mean-field arguments does not hold in general, and we derive a unifying general form for the effective uptake. Finally, we will show that several macroscale growth scalings are possible when microscale heterogeneity is present, even for simple microscale dynamics.

Taylor dispersion-controlled Rayleigh-Benard and Darrieus-Landau instabilities

Dr Prabakaran Rajamanickam

University of Manchester

Abstract:

The Rayleigh-Benard (RB) and Darrieus-Landau (DL) instabilities are two classical hydrodynamic instabilities induced by buoyancy and inertial forces that arises due to density variations caused by temperature variations in the fluid. Similarly, the notion of Taylor dispersion, also known as shear-induced dispersion, is a well-known concept of mixing promoted by an imposed shear flow in thin mixing layers. The coupling between the flow-induced mixing phenomena and the hydrodynamic instabilities aforementioned are studied within a two-dimensional Hele-Shaw configuration. As a preliminary important study, a systematic derivation of two-dimensional depth-averaged governing equations in thin mixing layers is carried out. The derivation is based on the lubrication approximation and accounts for variations in fluid and thermal properties, including density, viscosity, and thermal diffusivity. The resulting governing equations are then used to investigate the role played by flow-induced dispersion on the RB and the DL instabilities, both for horizontal and vertical mixing layers. The shear-induced dispersion occurs in the streamwise direction and not in the spanwise direction, whereas buoyancy-induced dispersion appears in both directions. In general, the flow-induced dispersion is found to thicken the mixing region either in an isotropic or anisotropic fashion, with corresponding implications on the two instabilities which will be discussed.

Predicting retinal haemorrhage following retinal vein occlusion

Miss Atrayee Bhattacharya

University of Glasgow

Abstract:

The retina is the layer of sensory tissue lining the back surface of the eye. Retinal vein occlusion is a blockage in the venous network, either in the main vessel (CRVO) or in one of the smaller veins (BRVO). Due to arteriosclerosis, the walls of arteries get damaged and swell. At points of arterio-venous crossing the retinal vein and artery share a common sheath and so swelling of the artery results in compression of the adjacent vein, narrowing the blood vessels. The flow profile reduces and produces venous thrombus and as a result occlusion of blood vessels occur.

We aim to understand how occlusion in one part of the network leads haemorrhage in other part of the network using a mathematical model (viscous and inertia model). We are looking how this process is influence by the wave propagation and pressure rises. To supplement the mathematical model we are using cutting edge fundus image analysis on clinical images. The idea is to validate the mathematical model based on the features extracted from images.

We observe that if the occlusion is not large enough or if we constrict the vessel slowly enough we get a viscous response. In contrast, if the perturbation is applied really quickly or the maximum amplitude is large enough then system exhibits wave propagation in the upstream of the constriction. For inertia model, we see a sharp front is moving and steepening of the profile is quite evident while viscous model responds quite smoothly.

2D Quasicrystal from self-assembled nanocolumns with polygonal cross-sections

Dr Xiangbing Zeng

University of Sheffield

Abstract:

Quasicrystals are intriguing structures that possess long-range positional order but no periodicity, typically with rotational symmetries that are "forbidden" in conventional periodic crystals. Here we report the discovery of the first two-dimensional columnar liquid quasicrystal, with dodecagonal symmetry (Nature Chemistry 2023, 15, 625–632). The structure consists of dodecagonal clusters made up of triangular, square and trapezoidal columns, self-assembled from T-shaped amphiphilic molecules. To maximize the presence of such locally optimal dodecagonal clusters, the system abandons periodicity but adopts a quasiperiodic structure following strict packing rules. The discovery opens an approach to creating other strict instead of random quasiperiodic structures in soft matter.

Bifurcation dynamics of the snapping of shallow circular arches

Mr Will Simpkins

University of Bristol

Abstract:

The ability of curved thin elastic systems to possess multiple stable configurations, as well as their ability to transition between these distinct configurations, has been of interest to researchers and engineers for many years. Although the quasi-static nature of these transitions has been widely studied, the dynamics of the transitions have received relatively little attention.

We consider a compressible bi-stable circular arch under a central load, and investigate its behaviour around the unstable equilibrium eventually reached by increasing the applied load. By performing linear and weakly nonlinear analyses, we find that the system can transition between stable configurations via symmetric and asymmetric buckling modes, and we show how these buckling modes originate from a variety of co-dimension one and two bifurcations.

From our analyses, we are able to determine how the geometrical parameters of the system not only affect the characteristics of the bifurcation but also alter the key dynamic quantities of the snapping, such as the time needed for the snapping transition to complete. These results provide new insights into how the shape of thin and curved structures can be controlled in engineering applications.

Diffusive Lotka-Volterra type systems: conditional symmetries, exact solutions and their properties

Professor Roman Cherniha

University of Nottingham - Institute of Mathematics of NAS of Ukraine

Abstract:

This talk is devoted to symmetries and exact solutions of the classical Lotka-Volterra system with diffusion and its highly non-trivial generalisation, the Shigesada-Kawasaki-Teramoto system. Lie and conditional (nonclassical) symmetries are identified and used for constructing exact solutions that satisfy typical boundary conditions and describe different scenario of population (cell, tumour) evolution as time tends to infinity. In particular, several highly nontrivial exact solutions, including travelling fronts, (quasi)periodic solutions and those with separation of variables are found, their properties are identifi ed and a biological interpretation is discussed.

The obtained exact solutions can also be used as test problems for estimating the accuracy of approximate and numerical methods for solving boundary value problems related to reaction-diffusion systems. Because the Lotka-Volterra type systems are used for mathematical modelling of an enormous variety of processes in ecology, biology, medicine, physics and chemistry, the talk could be interesting not only for specialists in PDEs but also for scholars from other branches of applied mathematics.

The talk is based on the results obtained in collaboration with John R. King (University of Nottingham) and Vasyl' Davydovych (Institute of Mathematics of NAS of Ukraine) and published in Commun. in Nonlin. Sci. and Num. Simul. 113 (2022), 106579, https://doi.org/10.1016/j.cnsns.2022.106579 and 124 (2023), 107313 https://doi.org/10.1016/j.cnsns.2023.107313.

The stability of moving fluid menisci in cylindrical geometries

Dr Paul Alexander

University of Manchester

Abstract:

A fluid plug can be supported in a narrow, vertical cylinder against the forces of gravity due to the contact angle and surface tension. In many applications, the plug moves dynamically within the cylinder, such as in a thermometer. This requires the treatment of a moving contact line, which entails additional modelling assumptions.

In this talk, I will present a numerical linear stability analysis of such a plug, both when it is static and travelling at a constant velocity, under variations in contact angle, velocity, the strength of gravity, and the slip length used in the contact line model.

The most unstable mode is typically non-axisymmetric (azimuthal Fourier mode one), which could not be detected in previous axisymmetric studies. The development of a non-axisymmetric plug propagation after the instability is of relevance for coating applications, in which a uniform film is typically desired.

I will discuss the contact line model and the non-standard finite element implementation, and present a detailed bifurcation analysis of the system that summarizes the stability analysis results in a compact form.

The slip length is found to have a significant effect on the stability in both static and dynamic situations, and this is likely to be the case in other problems involving moving films, such as flow down an inclined plane. It is possible that examining the stability in such a configuration experimentally could allow inference of the slip length.

Spectral approaches to stress relaxation in epithelial monolayers

Dr Natasha Cowley

University of Manchester

Abstract:

Vertex models are widely used in cell biology to model the complex mechanical interactions in epithelial cell sheets. This simple cell-based model with a small number of parameters has been successful in capturing tissue level phenomena. Understanding how the mechanics of cells drive changes in tissue morphology is particularly important in embryonic development where epithelia undergo a variety of deformations in order to form specific structures. Using the cell vertex model, we investigate the viscoelastic relaxation to equilibrium of a disordered planar epithelium. In its standard form, the model is formulated as coupled evolution equations for the locations of vertices of confluent polygonal cells. We use singular value decomposition to project modes of deformation of vertices onto modes of deformation of cells. We show how eigenmodes of discrete Laplacian operators provide a spatial basis for evolving fields. Decomposition of the spectrum of relaxation times into contributions from the eigenvalues of the Laplacian and interactions with the prestressed state allows us to consider the relative contributions of material and geometric stiffness to the dynamics of the epithelium.

A functional exchange 'shunt' in the umbilical cord: the role of coiling in solute and heat transfer

Miss Tianran Wan

University of Manchester

Abstract:

The umbilical cord is a crucial link between a fetus and its placenta. Although solute exchange in the placenta has been studied widely, exchange between the three helical blood vessels (two arteries and one vein) inside an umbilical cord has not been investigated. This study aims to quantify the impact of the cord's geometry on the inter-vessel transfer of solute and heat, which may affect the health and development of the fetus. A mathematical model was developed to characterise solute and heat exchange between coiled umbilical vessels. By solving steady diffusion within the cord tissue, we have evaluated the amount of solute and heat exchanged between the umbilical vessels, and explored how different vessel configurations within the cord affect 'shunting'. Our model predicts a stronger coupling between umbilical vessels in tightly coiled cords and in configurations where vessels are closer to each other. The model predicts that the limited variation and symmetry in human cords, measured using histology and ultrasound, could be interpreted as advantageous for fetal growth since it minimises 'shunting' between umbilical vessels.

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Flame propagation in channels: 2D and 3D flame dynamics

Mr Aiden Kelly

University of Manchester

Abstract:

Flame propagation and stability in two-dimensional channels are investigated with a focus on 2D and 3D diffusive-thermal (Turing) flame instabilities in a Poiseuille flow. This talk explores the effect of flow amplitude (Peclet number Pe), channel width (Damkohler number Da) and differential diffusion (Lewis number Le) on the stability of the flame, in both adiabatic and isothermal wall conditions. In the adiabatic case, steady flame solutions exist for all Le, Pe and Da, whereas in the isothermal case, they are limited by a minimum value of the Damkohler number. The steady solutions include symmetric and asymmetric (about the channel midplane) flames, flame tubes and others. Furthermore, multiple steady-state solutions are found to exist for a given set of parametric values. For the instability analysis, we focus on those steady solutions which are symmetric. The instability experienced by these flames appears as a combination of the traditional diffusive-thermal instability of planar flames and the recently identified instability corresponding to a transition from symmetric to asymmetric flames. Instability regions are identified in the parameter space for selected channel widths by computing the eigenvalues of a linear stability problem and are presented. In these instability regions, flames are found to transition to solutions including cellular flames, pulsating flames, and others. These are complemented by two- and three-dimensional time-dependent simulations describing the full evolution of unstable flames into the non-linear regime. A complete characterisation of these solutions is provided for a wide range of values of Le, Da and Pe.

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Kinetic models of many-particle systems with short-ranged inelastic interactions and clustering

Mr Calum Braham

University of Oxford

Abstract:

Models of physical systems as sets of particles interacting through generalised 'forces' are common to a wide variety of sciences. While individual-based (microscopic) models of such systems are generally conceptually simple, they are often intractable analytically and computationally as most systems to be modelled contain an infeasibly large number of particles. It is common in such circumstances to derive a mesoscopic kinetic model, which tracks the evolution of population-averaged probability densities through a PDE model over an individual particle's phase space. In this talk we develop a general method for deriving such kinetic models for second-order particle systems with short-ranged, inelastic interaction forces using matched asymptotic expansion in the small interaction-length parameter. We then extend our method to apply to 'clustering' systems, where particles' positions and velocities may be highly correlated post-interaction, a behaviour that would invalidate many of the typical assumptions made when deriving such continuum models. We use the Cucker-Smale velocity-averaging model of collective behaviour as a test case to evaluate our kinetic PDE model against individual-based particle simulations and compare the results to other common inelastic interaction models.

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Numerical Modelling of the Mixing Dynamics of Printed Micro-droplets

Mr Yatin Darbar

University of Leeds

Abstract:

This study explores the dynamics and mixing of droplets relevant to Inkjet Printing through simulations conducted using the Finite Volume Method toolbox OpenFOAM. Various applications such as the manufacture of bio-compatible materials, textiles, and electronics bio-compatible materials, textiles, and electronics rely on the adequate mixing of printed droplets. Despite a wealth of experimental studies using larger droplets to visualize internal flow, there's a lack of dynamical similarity in accounting for diffusion effects at inkjet scales. Our novel numerical method, employing the Volume of Fluid method, includes a transport equation for a diffuse scalar conserved within the droplet, allowing us to directly examine mixing within impacting and coalescing droplets. Numerical modelling also enables quantitative evaluation of mixing through a metric based on the standard deviation of the conserved scalar.

Our validated simulations reveal that molecular diffusion plays a surprisingly limited role in homogenizing inkjet droplets (approximately 50μm in size) within relevant time scales. Modelling at this scale poses a challenge since there is a large computation cost within the simulation framework chosen. This is addressed by using simulations at an intermediate scale between experiments and applications and deriving a matching for the diffusion of the conserved scalar in order to accurately capture the effects of diffusion in applications. With this matching, parameters relevant to printing such as scale and droplet separation can be explored to quantify which factors present methods to control droplet mixing.

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Biomimetic Soft to Hard Connectors

Mr Kit Simmonds

University of Bristol

Abstract:

If materials of different stiffness are connected at a surface and tractions are applied to the overall object then stress concentrations are likely to occur at the surface between the two materials. These stress concentrations can lead to fatigue and damage at the surface, potentially leading to separation of the two materials or other forms of failure. This type of failure means that it is difficult to develop robust systems that involve soft-to-hard connections between materials, especially when substantial forces will be applied through those soft-to-hard connections as they are in some soft robotics applications.

In nature, however, we find a range of effective soft-to-hard connections. Tendons, which are fibre-reinforced biological structures, serve as interfaces between hard and soft materials. These structures exhibit gradual transitions in stiffness to lessen stress concentrations, increasing the strength and reliability of the connection. For manufactured soft-to-hard connections we want to replicate this gradual transition in order to build a similar level of resilience.

In this talk, I will present work on developing and optimising a hyperelastic material model for a fibre-reinforced soft-to-hard connection that can mimic the spatially varying stiffness in the osteotendinous junction. The main focus of this work is on finding an optimal distribution of fibres, bulk modulus or shear modulus, within specified budgets that minimises the maximum shear stress experienced in a cylindrical fibre-reinforced connector under loading. Using particle swarm optimization and finite element simulations, we demonstrate the capability to minimise the shear stress experienced while taking into account multiple parameters. These include the applied strain, the fibre material properties, the overall fibre budget, the overall bulk or shear modulus budget, and the geometry of the connector.

The largely linear response of earth's ice volume to orbital forcing

Mr Liam Wheen

University of Bristol

Abstract:

Orbital forcing plays a key role in pacing the glacial-interglacial cycles. However, the mechanistic linkages between the orbital parameters — eccentricity, obliquity, and precession — and global ice volume remain unclear. Here, we investigate the effect of Earth's orbitally governed incoming solar radiation (that is, insolation) on global ice volume over the past 800,000 years. We consider a simple linear model of ice volume that imposes minimal assumptions about its dynamics. We find that this model can adequately reproduce the observed ice volume variations for most of the past 800,000 years, with the notable exception of Marine Isotope Stage 11. This suggests that, aside from a few extrema, the ice volume dynamics primarily result from an approximately linear response to orbital forcing. We substantiate this finding by addressing some of the key criticisms of the orbitally forced hypothesis. In particular, we show that eccentricity can significantly vary the ocean temperature without the need for amplification on Earth. We also present a feasible mechanism to explain the absence of eccentricity's 400,000 year period in the ice volume data. This requires part of the forcing from eccentricity to be lagged via a slow-responding mechanism, resulting in a signal that closer approximates the change in eccentricity. A physical interpretation of our model is proposed, using bulk ocean and surface temperatures as intermediate mechanisms through which the orbital parameters affect ice volume. These show reasonable alignment with their relevant proxy data, though we acknowledge that these variables likely represent a combination of mechanisms.

Nonlocal Hydrodynamics in Microparticle Acoustophoresis: Implications for Acoustic Tweezer Calibration

Mr Ricky Hunter

University of Bristol

Abstract:

Acoustofluidics is a rapidly growing field with applications in lab-on-a-chip devices, tissue engineering, and cell analysis. Despite the relevance of low Reynolds number hydrodynamics to acoustofluidic devices, there has been little connection between the two beyond Stokes' drag approximations. We aim to link these disciplines by investigating the role of nonlocal hydrodynamics in the context of microparticle acoustophoresis.

We use the method of regularized Stokeslets to model particle-particle and particle-boundary interactions in a spatially varying force field representative of a common acoustic tweezer. Our investigation spans various particle concentrations and boundary friction values to further understand these interactions.

Results show an increase in particle velocity when hydrodynamically coupled particles are close, as is the case in higher particle concentrations. Proximity to a boundary slows particles and reduces interparticle coupling impact. A simple 1D acoustic tweezer was constructed using rapid prototyping methods to collect experimental data for validation. Polystyrene particles were trapped, with their velocity determined by particle tracking.

Both experimental results and the addition of close boundary effects support the applicability of the standard Stokes' drag model for many acoustofluidic devices where particles sediment to the bottom of a channel. However, deviations occur in devices where particles are distant from boundaries and at high/low particle concentrations. In such cases, the Stokes' drag model may significantly over/underestimate forces applied by acoustic tweezers.
This study highlights the importance of nonlocal hydrodynamics in particle motion, emphasizing nuanced interactions that challenge simple corrective measures. Our findings offer insights for characterising acoustophoretic devices across diverse concentration and spatial configurations.

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Instabilities in falling thin liquid films laden with soluble surfactants above CMC

Dr Anna Katsiavria

Imperial College London

Abstract:

The linear stability of a thin falling liquid film which is laden with soluble surfactant above the critical concentration for the formation of micelles (CMC) and flows down an inclined plane is addressed in two dimensions. In the case of a clean liquid the critical Reynolds number is well-known to be a function of the inclination angle. It is also known that the presence of surfactants significantly affects the behaviour of the system through surface concentration gradients, which in turn induce Marangoni forces and attribute an elasticity to the interface, thus stabilising the film and increasing the critical Reynolds number. Insoluble and soluble surfactants below CMC have already been considered in the literature by addressing the equations for momentum and mass transfer coupled at the interface. In the latter case, the number of parameters involved increases, and the effects of surfactant solubility and sorption kinetics are non-trivial. The presence of micelles further complicates the system dynamics mainly through the kinetics of monomer aggregation and micelle dissociation which are coupled to the sorption phenomena. Analytical and numerical techniques are employed to explore the effects of this coupling on the critical Reynolds number.

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Models for long nonlinear longitudinal waves in elastic rods of variable cross section

Mr Jacob Vizor

Loughborough University

Abstract:

We revisit the derivation of model equations describing long nonlinear longitudinal bulk strain waves in elastic rods of variable cross section within the scope of the Murnaghan model. We first derive a bi-directional Boussinesq-type equation using the two small parameters present in the problem under the maximal balance condition. We then consider the case when the radius varies slowly compared to the wavelength and derive a uni-directional model in the form of a variable coefficients Korteweg-de Vries-type equation. We also discuss the ongoing experimental work on strain waves in Polymethyl-methacrylate (PMMA) bars and rods which has motivated the development of the reduced models.

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TIDA Neuron Bursting; Conductance modelling of a neuroendocrine population

Mr Jake Ahern

University of Bristol

Abstract:

Bursting is a multi-timescale neural activity pattern consisting of a fast action potential oscillation imposed upon a slower oscillation, leading to alternating periods of spiking and quiescence. Bursting in tuberoinfundibular dopaminergic (TIDA) neurons is thought to regulate the release of prolactin, a critical reproductive hormone. The mechanism underlying TIDA bursting currently remains unknown, although a calcium-activated potassium current (I_KCa) and a persistent sodium current (I_NaP) have been hypothesised to drive this activity. Furthermore, TIDA bursting is highly synchronised between cells and electrical synapses may support this synchrony. Here, a minimal conductance-based model is constructed and fit to experimental TIDA recordings. Both I_KCa and I_NaP are included in the model, alongside currents to generate action potentials. Techniques from nonlinear dynamics, including slow-fast analysis, are used to analyse a single-cell model and find a TIDA-like bursting solution. The model indicates that both I_KCa and I_NaP drive TIDA-like bursting by regulating the activity of the spike-generating currents, and experimentally verifiable predictions are made. Extending the model into a simple two-cell network, coupled with an electrical synapse, suggests that synchrony amongst the oscillators increases the network frequency when bursts are in phase, and reduces the network frequency when bursts are out of phase with one another. We relate our model to recent experimental findings and suggest protocols to further understand TIDA network activity.

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Semiclassical Trace Formula for Quantum Many-Body Model

Mr David Martin

Northumbria University

Abstract:

In quantum chaos, an important tool to study the correspondence between a quantum system and its classical limit is the trace formula. This relates the energy density of a quantum model to the properties of the periodic solution of its classical limit. Here we apply the trace formula to the Lieb-Liniger model for a finite number of particles. This model has played a central role in mathematical physics and, we aim to show, is particularly relevant for studying classical/quantum correspondence. The model deals with N 1-Dimensional Bose particles trapped in a box, interacting via a two-body potential chosen as the Dirac delta "function". We consider the repulsive large interaction strength regime in our derivations.

The original treatment by Lieb and Liniger uses the Bethe ansatz to find the energy spectrum and the corresponding eigenstates. The oscillating part of the trace formula will relate the quantum energy spectrum to the classical periodic trajectories. We test our trace formula in two ways. First by solving numerically the Bethe equations for N, computing a long sequence of levels to get their density and comparing with the asymptotic Weyl's law. Then comparing the obtained density with a sum over the periodic classical solutions.

Pattern formation in stochastic reaction-diffusion systems

Mr Fraser Waters

University of Bath

Abstract:

In recent work, we derived a complete classification of the simplest two-species mass-action reaction schemes which exhibit Turing instability. Each of these therefore has the potential to support stable patterned states.

A typical weakly nonlinear scaling reveals that, under suitable choices of reaction stoichiometry and for most classes of minimal scheme exhibiting spatially in-phase (""true activator-inhibitor"") Turing instability, stable patterns can indeed bifurcate supercritically from the spatially uniform state. Intriguingly, some classes of minimal scheme require different weakly nonlinear scalings since coefficients in the usual normal form vanish unexpectedly.

In the case of a subcritical bifurcation, we conjecture that the unstable branch of patterned solutions will switch back through a saddle node bifurcation to admit stable patterned states below the Turing instability threshold, and thus allow bistability between the zero state and finite-amplitude patterned states. If this bistability is sufficiently robust to finite-size effects, then this implies a distinctive mechanism for pattern formation in stochastic systems; via stochastic switching between stable patterned and non-patterned states.

Modelling the influence of the cellular microenvironment on cell cytoskeleton and adhesion development

Mr Gordon R McNicol

University of Glasgow

Abstract:

To function and survive cells sense and respond to their microenvironment through mechanotransduction, converting mechanical cues into a biological response. Focal adhesions (FAs), which bind the cytoskeleton to the surrounding extra-cellular matrix (ECM), are crucial to this process, serving as mechanosensers and intracellular signaling hubs. Contractile actomyosin stress fibres (SFs), a key component of the cell cytoskeleton, grow from these adhesions as they mature. In turn, these SFs exert forces on their attached adhesions, promoting their maturation and increasing signaling, closing a positive feedback loop. We present a new one-dimensional bio-chemo-mechanical continuum model to describe the coupled formation and maturation of FAs and ventral SFs in non-motile cells. We use a system of reaction-diffusion-advection equations to describe three sets of biochemical events: the polymerisation of actin and subsequent bundling into activated SFs; the formation and maturation of cell-substrate adhesions; and the activation of signaling proteins in response to FA and SF formation. We couple the evolution of these key proteins to a Kelvin-Voigt viscoelastic description of the cell cytoplasm and the ECM, facilitating calculation of the macroscale deformation. We employ this model to understand how cells respond to various external and intracellular cues in vitro, reproducing experimentally observed phenomena including non-uniform cell striation, cells forming weaker SFs and FAs on softer substrates and a minimum ECM ligand density below which FAs and SFs cannot form.

Stochastic prey-predator theory of the L-H transition in fusion plasmas -- time-dependent statistical analysis and information theory

Mr Patrick Fuller

Coventry University

Abstract:

The L-H transition is a novel phenomenon involving the sudden improvement in the plasma confinement when an input power threshold is reached. Notably, the transition is accompanied by the regulation between turbulence and sheared poloidal flows with certain causal relationship between this regulation and the L-H transition. Here, the poloidal flows are distinguished into means flows (driven by the background pressure gradient) and zonal flows (driven by turbulence). The qualitative behaviour of the L-H transition was previously predicted using a deterministic, prey-predator-type model for the amplitudes of turbulence, zonal flow shear, mean flow shear and the ion density gradient [1].

In this talk, we present new results from a stochastic L-H transition model by extending [1] to include stochastic noise terms in the dynamical equations for the turbulence, zonal flow shear and density gradient. The noise terms capture the effects of physics on much smaller temporal scales than the evolution of low frequency drift-like turbulence and large scale zonal and mean flows. We show how a time dependent probability density function (PDF) for the turbulence, zonal flow shear and density gradient evolves in time and provide new insight into the L-H transition that was impossible through the deterministic model. In particular, we show that the PDF is often strongly non-Gaussian and thus the mean value and other moments are of limited use. In addition, we also discuss the utility of information geometry [2] and other statistical measures in elucidating dynamic hysteresis associated with the L-H transition and its backward, H-L transition.

References:
[1] E. Kim et al, Phys. Rev. Lett. 90, 185006 (2003).
[2] E. Kim, J. Stat. Mech. 093406 (2021).

Spatially logarithmic simulations of Rayleigh-Benard convection at high Ra

Dr Curtis Saxton

University of Leeds

Abstract:

Turbulent convection at high Rayleigh numbers (Ra) involves active structures spanning orders of magnitude spatially, in the bulk and in the boundary layers. The required resolution can be expensive or impossible for direct numerical simulations (DNS). We consider alternative schemes based on logarithmic grids in Fourier space. Constant-coefficient linear operators are exact. Nonlinear operators are approximated by finite lattice-supported triads (somewhat akin to shell models, but enabling a range of nonlocal interactions). We apply these lattices to horizontal dimensions, combined with a Chebyshev representation vertically, to investigate the scaling properties for the heat transport in Rayleigh-Benard convection up to extreme Ra. As a side-effect of irrational k-space scaling, the spatial domain is horizontally infinite (not a periodic box) and populated by asymmetric, nonrepetitive flow structures. Thermal boundary layers are well resolved by the large Chebyshev resolution. We discuss details of these emergent phenomena, and present the global scaling laws. A few further methodological improvements could bring previously unattainable regimes into the reach of ordinary computer hardware.

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How plants ‘burst the bubble’ of the threat of embolisms

Mr Jared Carpenter

University of East Anglia

Abstract:

Plants transmit nutrients and signaling molecules via their vascular system. To successfully carry out such transport processes, plants must overcome a number of physical challenges. For instance, the transpiration-driven xylem system, transporting water from root to shoot, operates under absolute negative pressure. This absolute negative pressure is a metastable state which can induce nanobubble formation that is detrimental to the plant. Nanobubbles are bubbles that are usually between 50-200 nanometres in diameter that have the potential to form an embolism in the plant. Plants thus need to somehow balance physically dangerous conditions with their needs for nutrient acquisition, water transport and signaling. How plants meet these conflicting demands is currently unknown.

I will present current ideas and approaches for addressing this fascinating problem. One hypothesis is that surfactants, which vary surface tension on a bubble surface, in the xylem help break down these nanobubbles. My current research involves modelling a bubble in infinite space using Stokes flow and deforming its shape by perturbing its radius. We then determine under what conditions the bubble will remain in a stable state or will become unstable. I will present calculations that first use the assumption that the leading-order bubble radius is constant and that there are no surfactants present. My model is then refined to include the effects of surfactants.

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Multiscale modelling of hormone distributions regulating root development

Dr Kristian Kiradjiev

University of Nottingham

Abstract:

Hormones are involved in many developmental processes in plants, including growth; however, how they are distributed within plant tissues is not yet fully understood. Determining how cell-scale processes lead to tissue-scale patterns is key to understanding how hormones and morphogens are distributed within biological tissues and control development. In this talk, we present mathematical models for transport of a specific growth hormone, gibberellin (GA), within plant roots. We begin with a multi-cellular mathematical model for GA transport, in which we consider both passive and active transport via the NPF membrane proteins. In addition, we consider the effect of the subcellular vacuole compartment on the transport dynamics. The model predictions reveal how the clade of NPF transporters control the spatio-temporal GA distribution, and how transport into the vacuole affects GA diffusion, enabling GA to be stored for later use. Using an upscaled model, we describe GA transport along the whole root incorporating the metabolic pathway of GA and the conversion between different GA forms. Whilst focussing here on GA, our model is general enough to be applied to transport of other hormones and substances, and in different types of cells. Thus, our research demonstrates how mathematical multiscale modelling enables us to gain insights into experimental observations by determining how cell-scale processes control tissue-scale developmental patterns.

Reduction of quartz in silicon carbide reactors using a multiphase approach

Mr Brady Metherall

University of Oxford

Abstract:

We present a multi-phase model to study the reduction of quartz to silicon carbide in a laboratory-scale reactor. We model the transport of gases and solids, and the kinetics of the reactions involved in the reduction process. Through the analysis of the model, we aim to gain a better understanding of the underlying mechanisms driving the reduction of quartz and to identify key parameters that can be controlled to optimize the production of silicon carbide.

Abstracts of Mini-symposia

Discrete and continuum methods to describe cancer invasion processes

Dr Fiona McFarlane

University of St Andrews

Abstract:

The ability of biological cell populations to correlate their movement and proliferation processes can result in collective migration and evolving spatio-temporal patterns at the cellular population level. These collective mechanisms play an important role in the formation and growth of growing solid tumours. We have formulated an agent-based model of cancer invasion wherein the infiltrating cancer cells can occupy a spectrum of states in phenotype space, ranging from `fully mesenchymal' to `fully epithelial'. The more mesenchymal cells are those that display more migratory phenotypes, where we examine directed cell movement such as haptotaxis. However, as a trade-off, they have lower proliferative capacity than the more epithelial cells. We have then formally derived the corresponding continuum model, which takes the form of partial integro-differential equations for the local cell population density function. Despite the intricacy of these model, for certain parameter regimes it is possible to carry out a detailed travelling wave analysis and obtain invading fronts with spatial structuring of phenotypes. As such, the model recapitulates similar observations into the structures of invading waves into leader-type and follower-type cells, witnessed in an increasing number of experimental studies over recent years.

Data-driven modelling of reaction-diffusion patterns in synthetic biofilms

Miss Martina Oliver Huidobro

Imperial College London

Abstract:

Atomic spinor Bose-Einstein condensates have become prime testbed systems for the exploration of topologically complex defects and textures resembling those found in superfluid liquid He-3 and quantum field theories. They exist in a rich variety of magnetic phases with different order parameter symmetry support a wide range of different topological defects and textures, from quantised vortices to Skyrmions. Here I will show how spinor Bose-Einstein condensates are promising candidates for realisation of non-Abelian vortex dynamics. In media with point-group order-parameter symmetries, the topological charges of line defects may not commute. As a result, colliding vortices cannot reconnect without forming a connecting rung vortex. Such vortices have analogues from cosmology to liquid crystals. I will further demonstrate how analogues of magnetic monopoles exhibit a rich topological dynamics that include oscillations between spin-Alice-ring and split-core configurations.

References:

[1] M. O.Borgh and J. Ruostekoski, “"Core Structure and NonAbelian Reconnection of Defects in a Biaxial Nematic Spin2 Bose-Einstein Condensate"", Phys. Rev. Lett. 117, 275302 (2016).
[2] Y. Xiao, M. O. Borgh, A. Blinova, T. Ollikainen, J. Ruostekoski, and D. S. Hall, “"Topological superfluid defects with discrete point group symmetries"", Nat Commun. 13, 4635 (2022).
[3] G. Baio and M. O. Borgh, “"Composite cores of monopoles and Alice rings in spin-2 Bose-Einstein condensates"", arXiv:2401.04103.

Finding oneself among the stars: the importance of role models and active allyship for LGBTQ+ STEM professionals

Dr Claire Davies

University of Exeter

Abstract:

The phrase "you cannot be what you cannot see" is often used to emphasise the importance of role models for people from underrepresented and minority demographics. But how does one find people to look up to if the aspect of one's identity that one wishes to see reflected in others is not visible, and in some cases actively hidden? In this talk, I will focus on the importance of role models for lesbian, gay, bi, trans, queer, asexual and intersex (LGBTQ+) STEM professionals and students of STEM disciplines. I will draw on my experiences founding PRISM Exeter - a regional network for LGBTQ+ STEMM professionals in industry and academia - and detail how our community engagement work is helping improve LGBTQ+ students' sense of belonging in STEM subjects.

The conduit equation: hyperbolic approximation and generalized Riemann problem

Professor Sergey Gavrilyuk

Aix-Marseille University

Abstract:

The conduit equation is a dispersive non-integrable scalar equation modeling the flow of a low-viscous buoyant fluid embedded in a highly viscous fluid matrix (Lowman and Hoefer, 2013, Lowman {\it et al.} 2014, Maiden {\it et al.}, 2016, 2018, 2020, and references therein.) This equation can be re-written in a special form reminiscent of the famous Godunov's form proposed in 1961 for the Euler equations of compressible fluids. By exploiting such a structure, we propose a hyperbolic approximation of the conduit equation. The comparison of solutions to the conduit equation and those to the approximate hyperbolic system is performed : the wave fission of a large initial perturbation of a rectangular or of gaussian form. The results are in a good agreement. New generalized solutions to the conduit equation are discovered involving a finite assemblage of waves of the same period and linked with a constant solution by generalized Rankine-Hugoniot relations. Such a multi-hump structures interact which each other almost as solitary waves : they collide, merge and reconstruct after the interaction. Both, the exact and approximate hyperbolic system describe such an interaction with a good accuracy. This is a joint work with Boniface Nkonga and Keh-Ming Shyue.

The interplay between bulk flow and boundary conditions on the distribution of microswimmers in channel flow

Dr Smitha Maretvadakethope

Imperial College London

Abstract:

Biofilm formation impacts many fields, from medical technologies (e.g. catheter design) to infrastructure development (e.g. water supply pipes) due to contamination and infection risks. For the case of motile micro-swimmers, the early stages of biofilm behaviour are dependent on the physical properties of swimmers and their flow environments as these affect the likelihood of surface interactions and surface colonisation. In our work, we highlight the effect of boundary conditions on the bulk flow distributions, such as through the development of boundary layers or secondary peaks of cell accumulation in bulk-flow swimmer dynamics. For the case of a dilute swimmer suspension in 2D channel flow, we compare distributions (in physical and orientation space) obtained from individual-based stochastic models with those from continuum models, and identify mathematically sensible continuum boundary conditions for different physical scenarios (i.e. specular reflection, uniform random reflection and absorbing boundaries). We identify the dependence of the spread of preferred cell orientations on the interplay between rotation driven by sheared flows and rotational diffusion. We further highlight the effects of swimmer geometries, fluid shear, and the full history of bulk-flow dynamics on the orientation distributions of micro-swimmer wall incidence.

Full paper available in the Journal for Fluid Mechanics (DOI: 10.1017/jfm.2023.897)

Travelling waves in phenotypically structured models of cell migration into extracellular matrix

Miss Rebecca Crossley

University of Oxford

Abstract:

Collective cell motility is a widely observed phenomenon in many biological scenarios, particularly developmental biology and medicine.
A range of modelling approaches have been employed to study this, but they rarely take into account population heterogeneity, which is associated with treatment failure and the recurrence of cancerous tumours.

To model this, we consider a system of non-linear cross-dependent partial differential equations that represent the evolution of multiple populations in space and time, which take the form of reaction-diffusion equations. We first consider a homogeneous cell population where cells can degrade extracellular matrix (ECM), divide and move, before comparing this to a population consisting of 2 distinct cell phenotypes: where one has the ability to degrade the ECM and move and the other, to proliferate.

We show that the system displays multiple spatially homogeneous steady states and that the phenotypic structure of the invading cell population depends largely on the environmental and density dependence of the switching mechanisms of the cells.
Beyond providing insight into the qualitative behaviour of cell invasion models when cells are able to switch phenotypes, these models could also be used to guide future developments of treatments for related diseases.

Polar Fluctuations Lead to Extensile Nematic Behavior in Confluent Tissues

Dr Chiu Fan Lee

Imperial College London

Abstract:

Many properties of crystalline solids are heavily affected by lattice defects on the atomistic scale. To understand and compute these defect we consider atomistic models capturing both the defect core and the long-range elastic fields the defect creates. The structure of the long-range elastic field for a defect equilibrium is revealed by a rigorous far-field expansion, showing that it can be expressed as a sum of continuum correctors and discrete multipole terms both of which are computable. We develop a novel family of numerical schemes that exploit the multipole expansions to accelerate the simulation of crystalline defects. To enclose the simulation in a finite domain, a theoretically justified approximation of elastic multipole tensors is introduced, which leads to a novel moment iteration resulting in higher order boundary conditions. Prototypical numerical examples of point defects and dislocations are considered to show that our proposed numerical scheme matches the accelerated convergence rates in terms of computational cell size given by the rigorous convergence estimates.

The mechanical secrets of the squirting cucumber

Dr Finn Box

University of Manchester

Abstract:

Rapid movement is rare in the plant kingdom, but a prerequisite for ballistic seed dispersal.

The squirting cucumber (Ecballium elaterium) is an iconic example of bio-ballistics, widely known to explosively launch its seeds via a high-pressure jet.

Despite this, the exact mechanism of seed dispersal remains poorly understood. Here, through a combination of experimentation, high-speed videography, quantitative image analysis and a suite of mathematical models, we develop a full mechanical description of the process. We quantify the turgor pressure driving ballistic ejection, uncover key mechanical interactions between the fruit and stem both prior to and during seed ejection, and quantify and simulate the ballistic trajectories of seeds, which are dispersed to distances more than 1000 times their length. We demonstrate how these mechanical features contribute to a nearly uniform distribution of seeds away from the mother plant. Simulating over several generations provides a theoretical measure of the reproductive success of the plant; a comparative analysis with several hypothesized mutant plants then illustrates the particular efficiency of Ecballium, thus providing insight to the evolutionary adaptations of this unique plant.

Linking discrete and continuous models of cell birth and migration

Mr Duncan Martinson

University of Cambridge

Abstract:

Self-organization of individuals within large collectives occurs throughout biology, with examples including locust swarming and cell formation of embryonic tissues. Mathematical models can help elucidate the individual-level mechanisms behind these dynamics, but analytical tractability often comes at the cost of biological intuition. Discrete models provide straightforward interpretations by tracking each individual yet can be computationally expensive. Alternatively, continuous models supply a large-scale perspective by representing the "effective" dynamics of infinite agents, but their results are often difficult to translate into experimentally relevant insights. We address this challenge by quantitatively linking spatio-temporal dynamics of discrete and continuous models in settings with biologically realistic, time-varying cell numbers. Motivated by zebrafish-skin pattern formation, we create a continuous framework describing the movement and proliferation of a single cell population by upscaling rules from a discrete model. We introduce and fit scaling parameters to account for discrepancies between these two frameworks in terms of cell numbers, considering movement and birth separately. Our resulting continuous models accurately depict ensemble average agent-based solutions when migration or proliferation act alone. Interestingly, the same parameters are not optimal when both processes act simultaneously, highlighting a rich difference in how combining migration and proliferation affects discrete and continuous dynamics.

Riemann problem for polychromatic soliton gases: a testbed for the spectral kinetic theory

Dr Giacomo Roberti

Northumbria University

Abstract:

We use Riemann problem for soliton gas as a benchmark for a detailed numerical verification of the kinetic equation describing the evolution of the density of states in the nonlinear Fourier spectral phase plane. We construct weak solutions of the kinetic equation describing the collision of two dense, uniform soliton gases, each composed of a finite number of "quasi-monochromatic" components. We extract the macroscopic physical observables of the associated nonlinear incoherent wave fields (integrable turbulence) for the focusing nonlinear Schrodinger equations from the analytical spectral solution and compare them with the results of direct numerical simulations of respective polychromatic soliton gases. To numerically synthesize dense soliton gases we employ a novel method that combines advances in the spectral theory of the so-called soliton condensates and the recently developed effective algorithms for the numerical realization of N-soliton solutions with large N.

Interaction of soliton gas with variable mean flow

Dr Thibault Congy

Northumbria University

Abstract:

Inspired by the recent developments in the theory of soliton-mean flow interaction [1] we consider the interaction of a soliton gas with a simple hydrodynamic (rarefaction) wave initiated by a rapid jump of the initial data for the Korteweg-de Vries (KdV) equation.

This interaction results in the tunnelling through the rarefaction wave of certain families of the gas' solitons while other solitons remain trapped inside the wave. The evolution of soliton gas is described by the nonlinear integro-differential spectral kinetic equation which is appropriately modified to include the interaction with the simple wave background. The existing results on the individual soliton's interaction with slowly varying mean fields are readily generalised to the case of rarefied soliton gases while the interaction with dense gases requires a nontrivial development of the theory based on the properties of the so-called soliton condensates [2]. The obtained analytical results are compared with the numerical simulation of the KdV soliton gases propagating through simple hydrodynamic waves.

[1] M.J. Ablowitz, J.T. Cole, G.A. El, M.A. Hoefer, X-D. Luo, Stud. Appl. Math. 151, 795-856 (2023).

[2] T. Congy, G.A. El, G. Roberti, A. Tovbis, J. Nonlinear Sci. 33, 104 (2023).

The effect of gravity-induced shape change on the diffusion-limited evaporation of sessile and pendant droplets

Professor Stephen Wilson

University of Strathclyde

Abstract:

A comprehensive study of the effect of gravity-induced shape change on the diffusion-limited evaporation of thin sessile and pendant droplets on a horizontal substrate is performed. Specifically, theoretical predictions for the evolution, and hence the lifetime, of sessile and pendant droplets evaporating in four modes of evaporation, namely the constant contact radius (CR), the constant contact angle (CA), the stick-slide (SS), and the stick-jump (SJ) modes, are obtained. In particular, it is shown that gravity-induced shape change can cause quantitative differences in the evolution of sessile and pendant droplets compared to that of a droplet in the absence of (or in the neglect of) the effect of gravity. For example, while a zero-gravity droplet evolves according to the well-known “d^2" and "2/3" laws, an initially large sessile droplet evolves according to new "d" and "1/2" laws, and an initially large pendant droplet evolves with the contact radius and the volume (but not, of course, the contact angle) behaving as if the droplet was evaporating in the CR mode. Furthermore, it is found that for all four modes of evaporation a sessile droplet always evaporates faster, and hence has a shorter lifetime, than a zero-gravity droplet with the same initial volume, which in turn always evaporates faster, and hence has a shorter lifetime, than a pendant droplet with the same initial volume. It is also shown that the lifetimes of droplets evaporating in the SS and SJ modes both always lie between those of the same droplet evaporating in the extreme modes.

This is joint work with Dr Hannah-May D'Ambrosio, Dr Alexander W. Wray, and Dr Brian R. Duffy (University of Strathclyde).

Stress-shape misalignment in confluent cell layers

Dr Mehrana Nejad

Harvard University

Abstract:

We use the thermoacoustic Helmholtz equation to model thermoacoustic oscillations as an eigenvalue problem. We solve this with a Finite Element method. We parameterize the geometry of an annular combustor geometry using Free Form Deformation (FFD). We then use the FFD geometry, define the system parameters and impose the acoustic boundary conditions to calculate the eigenvalue and eigenvector of the problem using a Helmholtz solver. We then use adjoint methods to calculate the shape derivatives of the unstable eigenvalue with respect to the FFD control points. According to these gradients, we propose modifications to the control points that reduce the growth rate. We first demonstrate the application of this approach on the canonical example; Rijke tube. Then we extend the method to an academic combustor and lower the growth rate of the unstable circumferential mode. These findings show how this method could be used to reduce combustion instability in industrial annular combustors through geometric modifications.

From MathCity to MathsWorld - changing attitudes to maths

Dr Katie Chicot

The Open University

Abstract:

As the CEO of MathsWorldUK, Dr Katie Chicot has a strong interest in showing the many faces of maths to pupils and teachers. MathsCity, the first hands on maths centre in the UK, was founded in Leeds in October 2021 by MathsWorldUK. In MathsCity pupils and teachers can solve puzzles, discover hidden maths and explore ideas. In this session we'll look at the difference between problem solving and maths exploration and work to empower ourselves and our students to do both.

Delayed loss of stability of periodic travelling waves affects wavelength changes of patterned ecosystems

Dr Lukas Eigentler

University of Warwick

Abstract:

In magnetohydrodynamics, astrophysics and plasma physics, the most used quantifier of the magnetic field line topology is the magnetic helicity. We know that its presence restricts the dynamics of the fluid which would not be the case for a non-helical magnetic field of similar energy.

Various magnetic field geometries contribute to the magnetic helicity content, such as twisting, knotting, braiding and linking. Here I will present recent progress on relaxing magnetic field and their dynamics in plasmas, including work on twisted structures. The latter show a strong helicity creation and annihilation that can only be explained by taking into account the alignment of the magnetic field and the electric current density, which has implications on our understanding on the topology of large-scale magnetic structures.

How I learned to stop worrying and love the (Maths)Comm

Ms Kat Phillips

University of Bath

Abstract:

One of the most integral parts of research is the dissemination of knowledge, and being able to do this effectively can drastically improve the impact of your work. There are so many more routes of comunication that we as academics are able to utilise beyond the standard routes of writing papers and presentinc at conferences with talks and posters.This talk is an introduction to the breath of communication styles we will welcome in the rest of the minisymposia, as well as an exploration of the key concepts that are widely used by effective communicators.

Throughout my PhD I have developed a passion for maths communication, and started my outreach career on Twitch, a platform typically underused for STEM, but with an ever-growing community of communicators on the platform. This new medium of commuication forced me to discover what makes communication effective, and how I need to adapt if I want to keep the content the same while the medium may change. Through this somewhat unconventional method of outreach, I have developed a portfolio of tips and tricks that I believe can be utilised by academics at all stages of career to improve their communication methods.

Finally with this talk and minisymposia we hope to open the conversation about how we communicate as academics, as we believe there is no one right way to do things, however we hope that the discussions from this session will remain with you throughout your career.

Pattern formation by living droplets in chemoattractant gradients

Dr Philip Pearce

University College London

Abstract:

Gradients of extracellular signals organise cell behaviour in tissues. Although we have good models for chemotaxis of isolated cells in signal gradients, it is not clear how cells react to gradients when the cell population is undergoing 3D morphogenesis, in which cell-cell interactions and cell-signal interactions undergo extensive emergent behaviour. Using light sheet imaging to simultaneously monitor signalling, single-cell and population dynamics in Dictyostelium cell populations, we show that these cells migrate towards nutritional gradients in swarms. As the swarm advances, it deposits clumps of cells at the rear, triggering their differentiation. Clump deposition is explained by a model in which the cell swarms behave as living droplets, with cell proliferation and signal gradient remodelling opposing surface tension to promote droplet shedding. The model predicts vortex motion of the cells within the droplet emerging from active forces, which was validated by 3D tracking of single cells in the swarms. Our data and modelling show how the emergent dynamics of multicellular communities cause qualitative differences in chemotaxis from isolated or non-interacting cells.

Why are all geniuses predominately White males?

Professor Nira Chamberlain OBE

Loughborough University

Abstract:

Why are all geniuses predominately White males? Why are CEOs, Board members, Captains of industries, Corporate Leaders predominantly White males? The argument often presented in the defence of those who do the selecting is that many of the diverse applicants were simply not good enough or that certain diverse communities showed little desire to join the profession. This is the standard "Pipeline issue" conjecture which has become more of a self -fulling prophecy. However, the American business woman Leila Janah, had the belief that through effective Equality, Diversity, and Inclusion (EDI) strategies, the talent of underrepresented groups could be tapped. The alternative conjecture to the Pipeline issue is that "Talent is equally distributed, opportunity is not." In parallel with this, field of Data Sciences has grown dramatically in the past decade, providing Statistical EDI KPI that can detect and challenge practices, ideas and privileges that reinforce inequality. Nevertheless, in a meeting with a UK All-Party Parliamentary Group, a number expert witness argued that these KPIs did not reflect the lived experience of underrepresented groups. In this paper we will explore the idea of treating EDI as a pure science problem. In science, every problem has a baseline from which we can take measurements from. Hence by defining inequality as the numerical difference between talent and opportunity, we can scientifically measure the effectiveness of a company's EDI strategy. The method employed will be named as the Beyond Diversity Algorithm.

Preconditioned Iterative Methods for Time-Dependent Fluid Flow Control Problems

Dr John Pearson

University of Edinburgh

Abstract:

Spatial patterns are ubiquitous in nature, from chemical reactions on the molecular scale to zebrafish pigmentation on the cellular scale and clustering of vegetation in arid environments on the organismic scale. Surprisingly, a single mathematical mechanism, known as the Turing mechanism, is thought to underlie the formation of all these patterns. Due to its universality, the Turing mechanism has been hypothesised to explain the patterns in the spatial distribution of motile biological species, namely predators and prey. Yet, despite these predictions, such patterns have not been empirically tested or observed. In the talk, I will suggest an explanation for this lack of empirical evidence, proving that the evolution of organismic motility destabilises Turing patterns. While the evolution of predators and prey kills organismic Turing patterns, such patterns can play an important role at other scales of animal movement. In the second part of the talk, I will suggest how Turing patterns might emerge in the brain activity of moving animals and how such patterns can positively contribute to their fitness.

Dynamical density functional theory for active matter

Dr Michael te Vrugt

University of Cambridge

Abstract:

Dynamical density functional theory (DDFT) [1,2], a continuum theory that can be systematically derived from microscopic particle dynamics, is a central theoretical tool of soft matter physics. It has, in recent years, found a plethora of applications in the study of active and biological matter. In this talk, I provide a brief introduction to DDFT and give an overview over applications in active matter physics and biology. Moreover, I discuss mathematical challenges that arise in the application of DDFT to active systems, and how they can be overcome.

References:
[1] M. te Vrugt, H. Löwen, R. Wittkowski, Adv. Phys. 69, 121-247 (2020)
[2] M. te Vrugt, R. Wittkowski, J. Phys: Condens. Matter 35, 041501 (2023)

Spontaneous shape transformations of active surfaces

Professor Alexander Mietke

University of Oxford

Abstract:

Biological matter has the fascinating ability to autonomously generate material deformations via intrinsic active forces, where the latter are often present within effectively two-dimensional structures. The dynamics of such "active surfaces" inevitably entails a complex, self-organised interplay between geometry of a surface and its mechanical interactions with the surrounding. The impact of these factors on the self-organisation capacity of surfaces made of an active material, and how related effects are exploited in biological systems, is largely unknown.
In this talk, we will focus on active surfaces with broken up-down symmetry is broken, of which the eukaryotic cell cortex and epithelial tissues are highly abundant biological examples.
In such surfaces, a natural interplay arises between active stresses and surface curvature.
We demonstrate that this interplay leads to a comprehensive library of spontaneous shape transformations that resemble stereotypical morphogenetic processes. These include cell-division-like invaginations and the autonomous formation of tubular surfaces of arbitrary length, both of which robustly overcome well-known shape instabilities that would arise in analogue passive systems.

Quantifying invasive pest dynamics: the case of the oak processionary moth in the UK

Dr Laura Wadkin

Newcastle University

Abstract:

Invasive woodland pests are having a substantial ecological, economic, and social impact, harming biodiversity and ecosystem services. Mathematical modelling informed by Bayesian inference can deepen our understanding of the fundamental behaviours of invasive pests and provide predictive tools for forecasting the future spread. A key invasive pest of concern in the UK is the oak processionary moth (OPM). OPM was established in the UK in 2006, is harmful to both oak trees and humans, and its infestation area is continually expanding. Here, I will present a range of spatio-temporal modelling techniques applicable to describing the UK OPM infestation.

Geometrical optics stability analysis of rotating visco-diffusive flows

Dr Oleg Kirillov

Northumbria University

Abstract:

The geometrical optics stability analysis has proved successful in solving problems of ideal hydrodynamics and magnetohydrodynamics related to the stability of 3D flows of both compressible and incompressible fluids [1-3]. The method is based on perturbations of a background flow in a small parameter, representing a short wavelength. These perturbations are localized wave envelopes moving along the trajectories of fluid elements. Thus, detecting instabilities localized near a particular fluid element location, moving with the flow, reduces to solving a system of ODEs for the wave vector and amplitude, along particle paths in the underlying flow, with coefficients depending on the unperturbed velocity field [1-3]. Inclusion of viscosity or diffusivity in the analysis was long-time believed to be only stabilizing. However, starting with the works [4,5] that considered both viscosity and magnetic diffusivity in the geometrical optics analysis of helical magnetorotational instability it became possible to extend the method to visco-diffusive [6] and multiple-diffusive [7] flows for a wide range of Prandtl, Schmidt and magnetic Prandtl numbers. In the talk we review these works and discuss new applications.

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An asymptotic upscaling of transport through the bacterial membrane

Miss Molly Brennan

University College London

Abstract:

Transport through the outer membrane of many microorganisms, such as gram-negative bacteria, is often restricted to specific channels and non-specific porins. These provide a size-restricted, hydrophilic passageway for small molecules through an otherwise impermeable membrane. Important examples include antibiotics, which must cross the outer membrane to effectively target gram-negative bacteria, and quorum sensing molecules which allow bacterial colonies to coordinate mass phenotypic changes such as the production of virulence factors. In mathematical models, this limiting transport mechanism is often represented via constitutive boundary conditions. In this work, we systematically derive the correct effective boundary conditions to impose across the membrane in terms of physical channel and porin properties. We use a hybrid mathematical approach, combining multiscale methodology such as asymptotic homogenisation and boundary layer theory with numerical simulations. We also analyse the implications of the upscaled equations we derive in the context of real biological systems.

Electrostatic control of the Navier–Stokes equations for thin films

Dr Susana Gomes

University of Warwick

Abstract:

In this talk, I will present a robust control scheme for the Navier–Stokes equations modelling the two-dimensional multiphase flow of a thin film underneath an inclined solid surface. Control is exerted via the use of an electrode parallel to the substrate, which induces an electric field in the gas phase, and a resultant Maxwell stress at the liquid-gas interface. The imposed potential at the second electrode is derived using a Model Predictive Control loop, together with Optimal Control of a high-fidelity reduced-dimensional model. In this implementation, the interfacial shape of the fluid is successfully controlled, however the algorithm is sufficiently general to control any other quantity of interest.

Asymptotic models for vibration in asymmetric flexural structures: The generalised Rayleigh beam

Dr Michael Nieves

Keele Univeristy

Abstract:

The dynamic behaviour of asymmetric flexural systems, involving a master beam attached to a non-periodic collection of flexural resonators, is discussed [1]. The resonators couple longitudinal and flexural responses of the master beam. Its response is described via Green's functions, with intensities determined from an algebraic system embedding interactions of individual resonators. For infinite periodic waveguides, we derive an effective model called the generalised Rayleigh beam that supports flexural-longitudinal wave coupling and is efficient in regimes not typically encountered in homogenisation [2].

Acknowledgment: M.J.N. gratefully acknowledges the support of the EU H2020 grant MSCA-RISE-2020-101008140-EffectFact.

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An asymptotic solution for the evaporation of arbitrary-shaped droplets

Dr Madeleine Moore

University of Hull

Abstract:

Diffusion-limited evaporation of liquid droplets has received significant research interest due to its fundamental significance in a variety of industrial and engineering applications such as inkjet printing, microscale and colloidal patterning, DNA microarray technologies and the manufacture of Q/OLEDs. However, beyond circular and elliptical droplets, there are very few existing analytical solutions, despite a number of these applications exhibiting droplets of a more complex geometry. Here, we address this deficiency by deriving asymptotic results for the evaporative flux of a weakly non-circular droplet. We demonstrate how the framework may be extended to polygonal geometries and droplet arrays, with an eye to industrial applications.

Insights into the control of active matter

Dr Luke Davis

University College London

Abstract:

Active constituents burn fuel to sustain individual motion, giving rise to collective effects that are not seen in systems at thermal equilibrium, such as phase separation with purely repulsive interactions. There is a great potential in harnessing the striking phenomenology of active matter to build novel controllable and responsive materials that surpass passive ones. Yet, we currently lack a systematic roadmap to predict the protocols driving active systems between different states in a way that is thermodynamically optimal. Equilibrium thermodynamics is an inadequate foundation to this end, due to the dissipation rate arising from the constant fuel consumption in active matter. In this talk, I will present our versatile framework for the thermodynamic control of active matter. Combining recent developments in stochastic thermodynamics and response theory, our approach shows how to find the optimal control for either continuous- or discrete-state active systems operating out-of-equilibrium. Our results open the door to designing novel active materials which are not only built to stabilize specific nonequilibrium collective states, but are also optimized to switch between different states at minimum dissipation.

Adjoint based shape optimization for thermoacoustic stability of combustors using free form deformation

Mr Ekrem Ekici

University of Cambridge

Abstract:

Many patterned ecosystems, such as dryland vegetation patterns and intertidal mussel beds can be described by PDEs admitting periodic travelling waves (PTWs). Under a changing environment that increases stress, such systems undergo a cascade of wavelength changes before an extinction event occurs. Classically, wavelength changes have been predicted by identifying the intersection of a PTW's wavelength contour with a stability boundary in the system's Busse balloon. In this talk, I highlight that this information is often insufficient because of a delayed loss of stability phenomenon. I show that PTWs can persist as transients for ecologically significant times after the crossing of a stability boundary in the Busse balloon. I present a method that can predict the order of magnitude of the time delay between the crossing of a stability boundary and the occurrence of a wavelength change by linking the delay to features of the essential spectra of the PTWs.

Joint work with Mattia Sensi (Politecnico di Torino)

Data-driven inference of adjoint sensitivities without adjoint solvers: An application to nonlinear wave equations

Ms Defne Ege Ozan

Imperial College London

Abstract:

Adjoint methods offer a computationally cheap and accurate way to calculate the sensitivity of a quantity of interest with respect to all the system's parameters. However, adjoint methods require the implementation of an adjoint solver, which can be cumbersome. Furthermore, the accuracy of the adjoint solver relies on the physical model of the underlying system, which can be high-dimensional and nonlinear. Recently, data-driven methods, such as Echo State Networks (ESNs), have been shown to successfully learn nonlinear dynamics from data, including chaotic flows. In this work, we infer the parametrized nonlinear dynamics from data via a parameter-aware ESN to learn the adjoint sensitivity from the network. Specifically, we consider a prototypical model of a thermoacoustic system, which is governed by the coupling of linear acoustics with a nonlinear and time-delayed heat release. First, we derive the adjoint of an ESN, and compute the sensitivity of the acoustic energy with respect to the system parameters. Second, we improve generalizability and robustness by embedding the physical knowledge about the nonlinearity and the time-delayed nature of the thermoacoustic dynamics. Third, we employ the data-driven sensitivity provided by the adjoint of the trained ESN within a parameter optimization framework to minimise the acoustic energy. This work opens possibilities for data-driven gradient-based design optimization without adjoint solvers.

Vortices in supersolids

Dr Thomas Bland

University of Innsbruck

Abstract:

Since the first observation of a Bose-Einstein condensate (BEC) made from strongly magnetic atoms, these systems have proven to be a rich source of new and fascinating phenomena arising from the long-range and anisotropic dipole-dipole interaction. Recently, these dipolar quantum gases have proven to be a versatile platform in order to observe the long-sought after supersolid phase—a state that simultaneously manifests a crystalline order and superfluid properties. Seminal experiments in this burgeoning field have probed the superfluid nature of the dipolar supersolid, ranging from probes of the global phase coherence to investigations of the two Goldstone modes resulting from the breaking of phase and translational symmetries. In this context, I discuss the latest results from our erbium and dysprosium dipolar BEC labs in Innsbruck, reporting on the first experimental observations of quantized vortices in both the unmodulated BEC and supersolid phases, a hallmark feature of superfluidity. Finally, I theoretically investigate "glitches" in dipolar supersolids: instantaneous jumps of the rotation frequency occurring due to the angular momentum of ejected superfluid vortices being absorbed by a rigid crystalline component, akin to observations in neutron stars.

Projected models: short waves, thick films and nonlocality

Dr Alexander Wray

University of Strathclyde

Abstract:

Reduced order models, often derived under the assumptions of lubrication theory, are a critical method in modern fluid dynamics. They allow for dramatic simplifications of the governing equations, while still retaining the key physics inherent to the system. This can be used for everything from analytical insight to enabling large scale ensemble studies.

However, many systems are not amenable to lubrication theory, or diverge from reality in interesting limits such as thicker films, high levels of inertia, when waves are short, or when then intrinsic locality of a lubrication system is violated. A variety of approaches have been used in recent years to attempt to alleviate these issues, from models based on center manifold theory, to variational models, to projected-type models (including weighted residuals).

Projected-type models in particular have experienced a substantial amount of success in a variety of different, seemingly unrelated problems. Here we demonstrate the forefront of what is known to be possible, and synthesise these different threads into a more cohesive overview of the state of the art. Finally, we discuss open issues, and possibilities for the future.

High-order homogenisation of the time-modulated wave equation: non-reciprocity for a single varying parameter

Dr Marie Touboul

UMI 2004 Abraham de Moivre-CNRS, Imperial College London

Abstract:

Laminated media with material properties modulated in space and time in the form of travelling waves have long been known to exhibit non-reciprocity.

However, when using the method of low frequency homogenisation, it was so far only possible to obtain non-reciprocal effective media when both material properties are modulated in time, in the form of a Willis-coupling (or bi-anisotropy in electromagnetism) model. If only one of the two properties is modulated in time, while the other is kept constant, it was thought impossible for the method of homogenisation to recover the expected non-reciprocity since this Willis-coupling coefficient then vanishes. Contrary to this belief, we show that effective media with a single time-modulated parameter are non-reciprocal, provided homogenization is pushed to the second order. This is illustrated by numerical experiments (dispersion diagrams and time-domain simulations) for a bilayered modulated medium.

A model of cerebrospinal fluid flow around the brain

Dr Mariia Dvoriashyna

Univerisity of Edinburgh

Abstract:

Cerebrospinal fluid (CSF) is a clear, transparent fluid that bathes the brain and spinal cord. It fills the subarachnoid space (SAS), which surrounds the spinal canal and the brain and is externally lined by the dura membrane. During the cardiac cycle, this fluid pulsates in response to the time-varying changes in brain volume: during systole, CSF is displaced from cranial to the spinal compartment, and during diastole, the flow reverses. This flow plays a key role in mixing of solutes in the SAS.

In this study, we investigate the oscillatory CSF flow in the cranial SAS to comprehend its role in solute transport within the SAS and waste clearance. We develop a theoretical model of the flow using lubrication theory, as the SAS has a small aspect ratio. We impose realistic displacements of the brain surface obtained from MRI data [1]. The model reveals complex three-dimensional flow profiles within the SAS. We use this model to predict pressure gradients in SAS, which are challenging to measure experimentally.

The model also predicts the presence of a steady streaming driven by oscillations. This flow has a magnitude of approximately 100 m/s, comparable to another steady flow in SAS: the flow generated by the production and drainage of CSF. This suggests that steady streaming contributes to the mixing processes and transport of solutes within the SAS.

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A data-driven model of drug accumulation and expulsion dynamics in antimicrobial resistant bacteria

Dr Daniel Galvis

University of Birmingham

Abstract:

Antimicrobial resistance (AMR) presents a global health threat that could lead to over 10 million deaths annually by 2050. Moreover, the discovery of new antibiotic drugs has slowed significantly since the 1950s. Therefore, developing methods for rejuvenating existing antibiotic drugs is a key focus of experimental research into AMR. Recently, we developed a mathematical model of drug accumulation and expulsion in Salmonella enterica exposed to Ethidium Bromide (EtBr). We aim to use this model alongside experimental research to uncover the mechanisms underpinning this critical balance and to identify potential methods for promoting drug uptake in bacterial populations.

Through a combination of experiment and mathematical modelling, we systematically constrained a compartmental model of drug uptake dynamics in a bacterial population. Experimental datasets were acquired capturing drug accumulation dynamics under several conditions including efflux-pump inactivation, gene knockouts, and the growth phase of the bacterial population. Initially, we parameterised the model to the dynamics of drug accumulation where efflux pumps were inactivated. We then used this model as a baseline to study models for both a wild-type and knockout population through the parameterisation of additional nonlinear terms for drug expulsion dynamics. We found that we could capture key differences in the drug uptake over bacterial growth phases as variations in the membrane permeability of bacterial cells. This work suggests that the characterisation of efflux-pump protein kinetics could be invaluable for predicting whether a drug-rejuvenation therapy (e.g., modifying membrane permeability) will be effective in combatting antimicrobial resistance.

Understanding the early universe with Persistent Homology

Mrs Lina Julieth Castiblanco Tolosa

Newcastle University

Abstract:

The large-scale structure (LSS) of the universe has become an exceptionally good probe for the physics of the early universe. Upcoming LSS surveys have a strong potential for constraining primordial non-Gaussianity that provides information about the field content and dynamics during the inflationary era. However, extracting the maximum information content from observations is challenging. In the talk, I will introduce a new method that utilizes Persistent Homology to characterize LSS multi-scale topology and its sensitivity to the change in the initial conditions. Persistent Homology allows tracking the evolution of individual topological features (clusters of galaxies, filament loops, empty spaces) as they are created and destroyed by the continuous change in the length scales. Such that we can identify true features as the ones that remain invariant under the change of scale. Topological features are highly complementary to the standard statistical analysis relying on two-point correlation functions. The method has shown strong potential in extracting information from the LSS compared with other frameworks.

Pulse interactions driven by excited hidden modes

Dr Marc Pradas

The Open University

Abstract:

How do zebrafish get their stripes? Biologists have observed that different coloured pigment cells move around based on their interactions with each other. For example, yellow pigment cells chase black pigment cells, which then move away from the yellow cells in a roughly perpendicular anticlockwise direction. These cells can sense and interact with each other by extending protrusions which stretch over multiple cell-diameters. To investigate how long-ranged movement-inducing interactions can lead to self-organisation, mathematicians can use nonlocal advection-diffusion models. Such models, which are based on integro-partial differential equations, are used in developmental biology primarily to study cell-cell adhesion. They are also widely used in ecology to model the formation of swarms and territorial patterns, as both of these processes are driven by animals sensing each other over distances. However, current models only consider movement that is induced parallel to the separation of the interacting agents. What happens when the black zebrafish pigment cells move perpendicularly away from the yellow pigment cells? What is the function of this chiral 'run-and-chase' mechanism?

To address these questions, we extend nonlocal advection-diffusion models to incorporate chiral movement, and investigate how this affects pattern formation. Using linear stability analysis and numerical simulations, we demonstrate that chirality can help to promote pattern formation, whilst simultaneously allowing 'run-and-chase' mechanisms to separate cell types within the pattern.

Viscous fingering of unconfined thin-film flows

Mr Haolin Yang

University of Glasgow

Abstract:

It is well known that viscous fingering instabilities occur as one fluid intrudes into another in confined environments, but what if that confinement is removed? Rather surprisingly, we find that these instabilities still occur. In particular, we find that thin films of fluid spreading over a lubricated substrate are susceptible to a viscous fingering instability. Such flows are relevant to a range of geophysical, industrial and physiological applications from the small scales of coating applications to the large scales of magma flows. We apply lubrication theory to model such a flow of two thin films of viscous fluid assuming that they are driven by gravity and resisted by viscous shear stress, and that the effects of inertia and surface tension are negligible. We find conditions under which the flow is self similar and we examine similarity solutions across a number of regimes in terms of key dimensionless parameters, including the ratio of the viscosities of the two layers of fluid, the dimensionless density difference between them, and the dimensionless source fluxes. Depending on the viscosity ratio, the behaviour of the intruding layer ranges from that of a thin coating film, which exerts negligible traction on the lubricated substrate, to a very viscous gravity current spreading over a low-viscosity layer. We find that only the former flows are susceptible to instability, and we fully characterise the parameter space for which these instabilities occur.

From wave turbulence to integrable turbulence and soliton gases

Professor Stéphane Randoux

Université de Lille

Abstract:

The study of the nonlinear propagation of random dispersive waves has been an active research field in nonlinear physics since the 1960s. Historically, a significant portion of the research in this area has concentrated on weak wave turbulence. Wave turbulence theory (WT) examines the non-equilibrium statistics of incoherent and weakly nonlinear dispersive waves in non-integrable systems. Conversely, many physical systems are primarily described by partial differential equations (such as the nonlinear 1D Schrodinger equation or the Korteweg de Vries equation) that possess integrable properties, meaning they can be solved using the inverse scattering transform (IST) method.

Nowadays, the theoretical investigation of nonlinear random wave fields in integrable systems is addressed in the framework of 'integrable turbulence,' a concept introduced by Zakharov in 2009. A central element of integrable turbulence is the concept of soliton gases, which refers to large ensembles of solitons with random characteristics. In this presentation, I will discuss recent experimental findings related to integrable turbulence, with a specific focus on the topic of soliton gases propagating in various physical systems.

Optimal design of odd elastic metamaterials

Dr Jack Binysh

University of Amsterdam

Abstract:

Non-reciprocal interactions in active elastic media cause work cycles and wave propagation forbidden in equilibrium. These linear phenomena offer a route to designing autonomous materials that spontaneously crawl, roll or swim. Yet these same phenomena also render non-reciprocal materials hard to design, and force us to reckon with active elastodynamics beyond the linear regime.

In this talk I will describe our current work on rationally designing non-reciprocal materials made of robots, and modelling their collective dynamics. I will present a continuum model of nonlinear odd elasticity, benchmarked against microscopic simulation and table-top experiments. Combining non-reciprocity and non-linearity in momentum-conserving materials yields long-wavelength instabilities and travelling nonlinear patterns. Strikingly, momentum conservation causes these emergent patterns to coarsen over time. As a result, these active metamaterials spontaneously rid themselves of disorder in favour of coherent motion. We then explore how this coarsening can respond to environmental stimuli, leading to a toolkit of distinct patterns for designing locomotion and actuation.

Diffraction Theory and Several Complex Variables

Dr Valentin Kunz

University of Cambridge

Abstract:

We will discuss some new developments on linking diffraction theory -- the mathematical study of boundary value problems to the Helmholtz equation -- with the theory of several complex variables -- an area of mathematics extending the classical theory of functions involving a single complex variable to higher dimensions.

Ultrasonic travel-time tomography of locally anisotropic media using stein variational gradient descent

Dr Katy Tant

University of Strathclyde

Abstract:

In ultrasonic non-destructive evaluation, imaging defects embedded in metal, polycrystalline welds presents a significant challenge due to the presence of anisotropic grain structures which can cause the probing elastic waves to scatter and refract. The industrial gold standard for detecting and characterising defects embedded in these welds is the total focussing method (TFM) which, in its simplest form, incorrectly assumes a constant wave speed throughout the weld domain, and can thus lead to unreliable defect reconstructions. However, if information on the distribution of anisotropic crystal orientations present in the structure is known a priori, this can be used to correctly focus the ultrasonic waves and a significant improvement in flaw detection and accuracy can be obtained. Crystal orientation mapping using travel time tomography has been already been examined using Markov Chain Monte Carlo (MCMC) methods, however due to the high dimensional parameter space inherent to these non-linear tomography problems, these methods are computationally very expensive. Variational Bayesian Inversion (VBI) approaches can offer more efficient methods of inversion for these problems as they use deterministic optimisation techniques instead of relying on random sampling. In this work we apply the Stein Variational Gradient Descent approach to the anisotropic tomography problem, utilising a novel method for travel-time field calculations and ray tracing in heterogeneous and anisotropic media.

Twisted magnetic knots and links and their Current alignment

Dr Simon Candelaresi

University of Stuttgart

Abstract:

The tumour microenvironment is a complex system with many interacting cell populations which support or attack tumour cells. Adding to the complexity, altered behaviour of fibroblasts, a cell type responsible for producing extracellular matrix (ECM), creates a dense network of ECM which inhibits mobility of tumour fighting immune cells. These modified fibroblasts also assist the tumour in remodelling surrounding blood vessels, causing chaotic blood flow and hypoxia.

The aim of our research is to understand how vessel remodelling impacts the spatial localisation of cells in the tumour microenvironment, and the effect this has on a tumour's survival and sensitivity to treatment. We approach this problem by creating an off-lattice agent-based model of a tumour, focusing on mechanisms of vessel and matrix remodelling. Our model builds upon a tumour spheroid simulation, adding simplified mechanisms of vessel and matrix remodelling to create a minimal description of the tumour microenvironment. Building our model from this foundation makes it modular and flexible, allowing us to analyse the individual effect of each mechanism. The individual cell resolution from agent-based modelling is particularly suited to study the spatial localisation of cells in the tumour. Immunofluorescence and immunohistochemistry stained microscopy images are of similar resolution to our model, allowing us to validate our model against real tumours.

Key questions we address with our model include investigating how vessel remodelling changes a tumour's sensitivity to treatment, especially radiotherapy and immune control. Hypoxic regions created when vessels become occluded cause tumour quiescence which increases resistance to radiotherapy, and also induces processes which increase cancer cell motility, both of which result in poor patient prognosis. In future work we will incorporate T-cells, immune cells which directly kill tumour cells, into our simulation. We will use our model to investigate the effectiveness of the immune system in controlling tumours, focusing on the effects of T-cell exhaustion, a process in which T-cells become inactivated within the tumour microenvironment.

The 65th British Applied Mathematics Colloquium

Cold-atom analogues for vacuum decay

Dr Alex Jenkins

University College London

Abstract:

False vacuum decay plays a vital role in many models of the early Universe. However, we lack a satisfying theoretical understanding of this process, with existing approaches working only in imaginary (Euclidean) time, and relying on crucial assumptions that have yet to be empirically tested. An exciting route forward is to use cold-atom systems which undergo first-order phase transitions that are analogous to vacuum decay. In this talk, I will present recent theoretical work to understand this analogy using semiclassical lattice simulations, and will discuss possibilities and challenges for realising these analogues in the laboratory.

Nonlinear wave-particle interactions in near-earth space: what makes our magnetosphere sing?

Dr Daniel Ratliff

Northumbria University

Abstract:

Our planet, like several others, is surrounded by a plasma trapped within the Earth's magnetic field forming an environment known as a magnetosphere. Whistler-Mode Chorus (WMC) waves are an important contributor to the dynamics of this magnetosphere, not only for their prevalence in measured observations of near-Earth space but also for their dominant role in transporting energy and particles throughout it. It is therefore of key importance to space weather modelling that we understand the phenomenology of WMC waves. There are also fundamental physics question to answer, as WMC waves display nonlinear phenomena rarely seen in other fields. Most famously, it is their ability to raise and lower their frequency repeatedly and rapidly leading to rising and falling tone waves respectively. Are the interactions between the wave and the particles driving such phenomena, and if so to what degree are they doing so?

In this talk, we revisit the nonlinear evolution of WMC waves from a theoretical perspective. Wave-particle interactions are shown to be a key driver of the modulational instabilities that lead to packet formation, which are well represented by an extension of the Nonlinear Schrodinger equation. Simulations of this yields power spectrum reminiscent of the rising and falling tone emissions observed in mission data from the satellite data and determines that that wave-particle interactions are the primary cause of this effect. As a result, this nonlinear theory indicates regimes in which these frequency sweeps can be enhanced or dampened, and the potentially dangerous consequences on particle energisation.

Multiple-scale asymptotic modelling of cold sheet metal rolling

Ms Mozhdeh Erfanian

University of Warwick

Abstract:

Rolling is a well-established metal forming process where a strip of metal becomes thinner by passing between successive pairs of rotating rollers. Mathematically, the governing conservation equations for mass and momentum are complicated by the nonlinear elastic–plastic constitutive laws. While a full analytical solution to the problem is impossible, here we simplify the problem to allow semi-analytic solution.

We apply a multiple scales asymptotic analysis, resulting in a 1D computationally tractable ODE to be solved, coupled with a Burgers equation first order correction. The asymptotic analysis exploits the small friction coefficient and the large aspect ratio related to the long and thin roll gap, both relevant to industrial cold rolling practice.

With a perfect plastic material assumption, this model successfully predicts the inhomogeneity in stress and velocity distribution for a rigid plastic material, including through-thickness stress oscillations, and the formation of an expansion fan and its subsequent development. While the model is validated against finite element simulation, it also serves as a benchmark for correct simulation. Furthermore, the model's precision, simplicity, and speed of calculation make it promising both as an on-line model for designing pass schedules and an off-line model for the optimization of final product properties.

Asymptotic analysis of wrinkles in a floating elastic sheet

Dr Anthony Bonfils

University of Limerick

Abstract:

We examine the buckling shape and critical compression of a confined sheet floating on a liquid. From the Föppl–von Kármán model of Hookean elasticity, we formulate a fourth order eigenvalue problem where the eigenvalue is the compressive load imposed on the sheet and the eigenfunction is the midplane displacement. A homogeneous sheet exhibits a degeneracy of wrinkled states for certain sheet's sizes; we explain this degeneracy using an asymptotic analysis valid for large sheets, and show that it corresponds to the switching of the sheet between symmetric and antisymmetric buckling modes. This degeneracy disappears when adding liquid inclusions, such that the bending stiffness varies parallel to the direction of confinement. Then, we study the concentration of the bending energy towards the soft regions of the sheet using a WKB method.

Reference: Soft Matter, 2023, 19, 8729

Interactions between control and modelling for deformable bubbles

Dr Alice Thompson

University of Manchester

Abstract:

As applied mathematicians, we often aim to understand physical problems by formulating a model and analysing its solutions. Ideally the model would be in good agreement with reality and its predictions largely correct, so that inferences can be made about control and optimization. But how do we assess the performance of the model itself? We can compare observations from experiments to model predictions, but usually only for stable states. Unstable states can be dynamically important but are usually harder to access in experiments. In this talk, I discuss how feedback control, in the guise of control-based continuation, can be used to detect and stabilize unstable states in experiments, without necessarily requiring a model at all. I will illustrate within the context of deformable bubbles propagating in a fluid-filled Hele-Shaw cell. We work with a depth-averaged model as the basis for numerical experiments, and show how CBC can be used to detect and stabilize unstable solution branches. Modifying our setup to allow for bubble propagation relative to fixed actuators introduces new complexity to both bubble control and control-based continuation. I end by a discussion of how easily CBC transfers to real physical experiments for both bubbles and for nonlinear soft matter systems more generally.

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The thermalisation of light in a photonic mesh lattice

Mr Tommy Moorcroft

University of East Anglia

Abstract:

A photonic mesh lattice is a unique optical configuration wherein both spatial and temporal coordinates are discretized. Experimental observations reveal that, as light traverses the lattice, energy tends to reach a thermalized state across all Fourier modes, facilitated by nonlinear wave interactions for a closed system. The dynamics of the photonic mesh lattice are fully described by discrete-time quantum walk equations. In this discussion, we explore the theoretical framework under the assumption of weak nonlinearity, demonstrating that energy movement in Fourier space is governed by a four-wave interaction. Employing weak wave turbulence methods, we establish that energy thermalization occurs across all Fourier modes. Furthermore, we provide a prediction of the timescale required to achieve thermal equilibrium, given as a function of the strength of nonlinearity.

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Chalkdust, VisualPDE, and other ways of making your mathematical life more fun

Dr Adam Townsend

Durham University

Abstract:

Enhance your mathematical life by telling people what you're up to. In 2015, a bunch of us as PhD students started Chalkdust, a magazine for the mathematically curious, as a way to tell stories and jokes about maths. Now at Issue 19 and shipping to many universities, I'll talk about how we put a fun maths magazine together and reveal some of its secrets. I'll say what we think makes a great article or a great topic, and show how outreach can remind us of the things we really like about the subject, as well as making us better at our Real Jobs. I'll also talk about VisualPDE, a new project to bring some of the excitement of PDEs (honest) to the world through the latest web tech.

Instability and transition in high Reynolds number flows with constant throughflow

Mr Jamie Cuthbert

Imperial College, London

Abstract:

There are many situations where a fluid may experience a throughflow, such as in porous geological formations. Throughflows can also be of use in industrial applications, such as reducing drag around the aerofoil of a plane since they can delay the transition from laminar flow to turbulent flow. In this talk, the impact of a constant throughflow will be considered on two parallel flows, Poiseuille flow and Couette flow. Both the linear and non-linear stability will be considered and an asymptotic analysis using critical layer-wall layer interaction will be shown to determine self-sustaining solutions at asymptotically large Reynolds numbers. This analysis shows that increasing throughflow does stabilise the fluid for a greater range of Reynolds numbers, but also expands the range of wavenumbers over which instability occurs. It is found that sufficiently strong throughflow will destabilise Couette flow in both the linear and non-linear regimes. Furthermore, parity is shown between the limits of the linear regime and non-linear regime, and a numerical simulation of the Navier-Stokes equations is undertaken to verify the results at finite Reynolds numbers.

Persistent and anti-Persistent Motion in Bounded Space: Resolution of the First-Passage Problem

Mr Daniel Marris

University of Bristol

Abstract:

The presence of temporal correlations in random movement trajectories is a widespread phenomenon across biological, chemical and physical systems. While persistence may be beneficial to reduce the oversampling of regions of space during search processes, e.g. in foraging organisms, anti-persistence may favour the continuous proximity of multiple agents, e.g. in enzyme catalysis to favour reactant interactions.

Despite a substantial body of work on persistent motion, little progress has been made in determining the dynamical properties of various transport-related quantities, including the first-passage or first-hitting probabilities to one or multiple absorbing targets in bounded space. To bridge this knowledge gap we generalise the first-passage, and splitting probabilities to be valid in arbitrary dimensions with partially absorbing targets for the one step non-Markovian extension of the lattice random walk, the correlated random walk. The discrete formalism allows us to consider both persistent and anti-persistent motion in hypercubic lattices as well as in the hexagonal lattice and to extend the notion of first-passage to that of the directional first-passage, whereby the walker must reach a target from a prescribed direction for a hitting event to occur.

We uncover the existence of multiple peaks in the first-hitting probability dynamics with reflecting domains and as an application to spatio-temporal observations of correlated moving cells that may be either repelled or attracted to hard surfaces, we compare the first-passage statistics to a target within a reflecting domain depending on whether an interaction with the reflective interface invokes a reversal of the movement direction or not.

Modelling the epidemiological implications for SARS-CoV-2 of Christmas household bubbles in England in December 2020

Dr Edward Hill

University of Warwick

Abstract:

In this talk I will summarise work originally conducted in late 2020 that was contributed to Scientific Pandemic Influenza Group on Modelling, Operational sub-group (SPI-M-O) of SAGE (Scientific Advisory Group for Emergencies) on Christmas household bubbles in England. In November 2020, plans were published to allow individuals to socialise within 'Christmas bubbles' with friends and family. This policy involved a planned easing of restrictions in England between 23–27 December 2020, with Christmas bubbles allowing people from up to three households to meet throughout the holiday period. Using a household model and computational simulation, we estimated the epidemiological impact of both this and alternative bubble strategies that allowed extending contacts beyond the immediate household. We provide this account as an illustration of a real-time contribution of modelling insights to SPI-M-O during the COVID-19 pandemic.

Clumping in flow: discrete to continuum modelling of magnetic particle transport

Dr Edwina Yeo

University College London

Abstract:

Magnetic nanoparticles are biocompatible particles whose trajectory can be controlled through the application of an external magnetic field. This remote control has applications in non-invasive drug delivery methods, cancer therapy and medical imaging. In vivo, magnetic nanoparticles are transported through the bloodstream and experience mechanical forces from the blood and external magnetic fields. Additionally, the nanoparticles experience attractive particle-particle interactions. This can adversely affect the efficacy of therapies for cancer treatment, or result in potentially dangerous aggregation of drugs in the bloodstream during magnetic drug delivery.

In this talk, we will present a novel mean-field model for the transport of magnetic nanoparticles including interparticle magnetic forces. This continuum model allows us to predict flow and particle density regimes where aggregation can be mitigated. We will conclude by comparing this model to discrete numerical simulations to demonstrate the successes and limitations of this mean-field approach.

How does ECM stiffness affect spheroid growth?

Ms Margherita Botticelli

University of Birmingham

Abstract:

When a tumour develops in a primary organ, the cancer cells can invade and migrate collectively. This can lead to metastasis, which is the primary cause of death in cancer patients. Collective cell migration is strongly influenced by how cells interact with each other and with the surrounding environment, which includes the extracellular matrix (ECM). Mathematical models can be used to study and predict the dynamics of collective cell migration. A type of model commonly used in cell migration is a hybrid discrete-continuous model, which couples a discrete agent-based model for the cells with a continuum model for the microenvironment. The model we want to build is based on the underlying biology of in vitro experiments in 3D, with the aim to identify how the stiffness of the extracellular matrix affects the growth of a spheroid of cancer cells in 3D. We are developing the model in PhysiCell, an open-source agent-based modelling platform which implements an off-lattice, centre-based model for the cells, together with a PhysiCell ECM extension to model the matrix.

Local systems for periodic data

Mx Adam Onus

Queen Mary University, London

Abstract:

Periodic point clouds naturally arise when modelling large homogenous structures like crystals. They are naturally attributed with a map to a d-dimensional torus given by the quotient of translational symmetries, however there are many surprisingly subtle problems one encounters when studying their (persistent) homology. It turns out that bisheaves are a useful tool to study periodic data sets, as they unify several different approaches to study such spaces. The theory of bisheaves and persistent local systems was recently introduced by MacPherson and Patel as a method to study data with an attributed map to a manifold through the fibres of this map. The theory allows one to study the data locally, while also naturally being able to appeal to local systems of (co)sheaves to study the global behaviour of this data. It is particularly useful, as it permits a persistence theory which generalises the notion of persistent homology. In this talk I will present recent work on the theory and implementation of bisheaves and local systems to study 1-periodic simplicial complexes. Finally, I will outline current work on generalising this theory to study more general periodic systems for d-periodic simplicial complexes for d>1.

Diversifying the Maths Curriculum at Queen Mary University of London

Mx Adam Onus

Queen Mary University, London

Abstract:

The history (and present-day state) of mathematics is a complex piece of theatre with many characters, each interacting with their contemporaries and their predecessors. Mathematics, as it is traditionally taught, involves recounting major events of this play, restricting the cast to a narrow list of figures. We can fight this stigma with increased awareness of diverse representation; not just with people of diverse backgrounds but also people applying their skills in diverse ways in academia and industry. Various members of the QMUL mathematics community sought to address this with cocreation projects over the 2022 and 2023 summers. Role models influence and shape our future, which led us to compile resources about mathematicians, past and present, who, despite their contributions to the field, are often left out of everyday mathematical discourse due to systemic and historic prejudices, discrimination and oppression. In this talk I will discuss the educational resources we have developed and are currently integrating into our undergraduate curriculum, as well as our recent and ongoing outreach efforts to spread our resources on social media and at secondary schools.

Multiparameter persistence for spatial biology

Ms Katherine Bejamin

University of Oxford

Abstract:

Single-parameter persistent homology – the flagship tool in topological data analysis – has witnessed a wide range of successful applications in the biological sciences in the last decade. Multiparameter persistent homology is a natural generalisation allowing for higher-order analysis of more complex phenomena including time-varying data. In this talk, we demonstrate some applications of multiparameter persistent homology to spatial data in biology.

Matching Macrophage Mediated Metastasis Models

Dr Joshua Bull

University of Oxford

Abstract:

Interactions between a growing tumour and the surrounding microenvironment can be highly influential, determining the prognosis of the disease and the efficacy of treatments. One key component of this interplay is the role of the immune system, one of the primary mechanisms through which the body can fight back against cancer. While the immune system should provide a defence against disease, in cancer it can be hijacked to aid tumour growth instead of eliminating cancerous cells. Here, we focus on how tumours reprogram macrophages, a key immune cell type, to help individual tumour cells migrate towards nearby blood vessels and escape into the vasculature [1]. In previous work we developed a complex agent-based model of this system, which reproduces biologically expected behaviour but is difficult to parameterise and is computationally expensive [2].

In this talk, we present a system of compartmental ODEs which act as a surrogate model for the ABM. We show that these equations can replicate the behaviour observed in [2], but also predict new behaviour outside of the parameter regimes previously considered in the ABM. We further simulate treatment via inhibition of a key chemokine in the system, and show that in certain regimes this treatment is effective at preventing metastasis. Finally, we demonstrate how these findings can be translated back into the ABM framework, permitting the two models to work in tandem to better understand the biology of macrophage mediated tumour metastasis.

[1] Arwert, 2018. Cell Reports https://doi.org/10.1016/j.celrep.2018.04.007

[2] Bull, 2023. PLoS Computational Biology. https://doi.org/10.1371/journal.pcbi.1010994

Modelling phenotypic and spatial heterogeneity in solid tumours

Dr Giulia Celora

University College London

Abstract:

In cancer, treatment failure and disease recurrence have been associated with tumour heterogeneity, where cells with different traits (i.e., phenotypes) coexist. Mathematical modelling is an effective tool to understand how the tumour micro-environment drives intra-tumour heterogeneity and to identify effective treatment strategies. In this talk, I will discuss a structured-population model that describes the evolution of tumour heterogeneity in a slice of tissue which is oxygenated from the boundary by a vessel. The model consists of a system of coupled non-local and non-linear partial differential equations that link the phenotypic evolution of tumour cells to the local oxygen levels. By combining dynamic simulation and bifurcation analysis, I will show that the inclusion of phenotypic heterogeneity drives the emergence of a strong Allee effect and how this shapes tumour growth dynamics.

History for Diversity in Mathematics: forming a network

Professor Isobel Falconer

University of St Andrews

Abstract:

The History for Diversity in Mathematics network aims to build a community and experience around using history of mathematics to promote equality, diversity and inclusion in the mainstream mathematics curriculum in UK Higher Education. It was prompted by outcomes of the annual Black Heroes of Mathematics meeting, and the 2-day workshop Is Mathematics Inclusive or Exclusive? Putting Colour, Culture and Context in the Curriculum (January 2022) which raised awareness of the potential use of history of mathematics in tackling EDI issues within mathematics.

So far, we have held focus groups with mathematics teaching staff around the UK, gathering baseline information around:

- In what ways are current HE maths curricula experienced as exclusive?
- What examples are there of using history to make the curriculum more inclusive?
- What support do practitioners need in using history to diversify the curriculum?
- How is the effectiveness of such initiatives best evaluated?

Focus groups with mathematics students are taking place in spring 2024. We are planning more network meetings, and an online reading group, and are developing a bibliography and literature review as the beginning of a resources collection.

This talk will report our findings so far, discuss some of the experienced benefits and pitfalls of using history of mathematics in this way, and outline future plans.

The network is funded by the Isaac Newton Institute and led by St Andrews, the Open, and Oxford Universities. For further information and notifications, join our JISCmail list history-for-edi-in-maths@jiscmail.ac.uk or visit the website https://mathshist4edi.wp.st-andrews.ac.uk/

The 65th British Applied Mathematics Colloquium

Drop rebound on a deep bath

Professor Paul Milewski

Penn State University

Abstract:

Small droplets can rebound from a fluid surface in a range of speeds. This is easily observed by looking closely at the secondary impact of raindrop splashes in a puddle. The rebound occurs due to the capillary restoring forces from the free surface being transmitted through the lubrication layer separating the droplet and the bath, providing sufficient vertical momentum to the droplet to lift it off the surface. We will derive a model capturing the fundamental physics for the simulation of this phenomenon and compare the results to experiments and simulations of the full fluid equations. We also consider the simpler case of the rebounding impact of small hydrophobic solid spheres. This is joint work with Kat Phillips and Radu Cimpeanu.

Stability of hydroelastic waves

Professor Emilian Parau

University of East Anglia

Abstract:

Two-dimensional periodic travelling hydroelastic waves on water of infinite depth are investigate. The stability of these periodic waves is examined using a surface-variable formulation in which a linearised eigenproblem is stated on the basis of Floquet theory and solved numerically. This work is joint with Mark Blyth (UEA) and Zhan Wang (CAS).

Quantized Vortices in Fuzzy Dark Matter Halos

Dr Gary Liu

Newcastle University

Abstract:

Fuzzy dark matter (FDM), composed by hypothesis ultralight bosons with de Broglie wavelength in galactic scales, exhibits quantum wavy features and the quantum pressure can against the gravitational collapse with gaining attention to researchers. FDM simulations have shown the potential to tackle the small-scale challenges in the successive cold dark matter model, such as missing-satellite, too-big-to-fail, and cusp-core problems. In this talk, we will discuss the characteristics of FDM halo structures, which not only introduce interference patterns in the dark matter density distribution and contain a compact solitonic core but are also a bed of quantized vortices, a type of topological defect in the context of superfluid and superconductor systems. The vortices in the halo are the source of density fluctuations, and the vortex energy and granule power spectra show that the inter-vortex distance and granule size are closely related. Lastly, we briefly discuss the role of self-interaction in the halo structures.

Beyond the Richards equation: two-phase flow in an unsaturated porous medium

Professor Michael Vynnycky

University of Limerick

Abstract:

Flow in an unsaturated porous medium is typically modelled using the Richards equation, which is unquestioningly believed to be an accurate enough approximation when the viscosity of the fluid being displaced, e.g. air, is much smaller than that of the infiltrating fluid, e.g. oil or water. Here, we apply asymptotic and numerical methods to a one-dimensional problem, consisting of the Buckley-Leverett equation, to show that this is not the case. With the viscosity ratio as a small parameter, we find that the Richards equation gives a leading-order solution that is not uniformly valid over the whole domain of interest. Instead, whilst the Richards equation holds for the bulk flow, the problem has a weak boundary layer for the saturation function, s, at the infiltration boundary, i.e. there is a boundary layer in the spatial derivative of s, but not in s itself. Although seemingly insignificant, this has a dramatic effect on the time taken to fill the porous medium: instead of filling exponentially quickly, it fills algebraically slowly. As a consequence, using the Richards equation will dramatically underestimate the time taken to fill a porous medium. Numerical computations are provided to underscore these asymptotic predictions.

“I’ll try spinning, that’s a good trick!” — how nonlocal chiral movement helps forms patterns

Mr Thomas Jun Jewell

University of Oxford

Abstract:

How do zebrafish get their stripes? Biologists have observed that different coloured pigment cells move around based on their interactions with each other. For example, yellow pigment cells chase black pigment cells, which then move away from the yellow cells in a roughly perpendicular anti-clockwise direction. These cells can sense and interact with each other by extending protrusions which stretch over multiple cell-diameters. To investigate how long-ranged movement-inducing interactions can lead to self-organisation, mathematicians can use nonlocal advection-diffusion models. Such models, which are based on integro-partial differential equations, are used in developmental biology primarily to study cell-cell adhesion. They are also widely used in ecology to model the formation of swarms and territorial patterns, as both of these processes are driven by animals sensing each other over distances. However, current models only consider movement that is induced parallel to the separation of the interacting agents. What happens when the black zebrafish pigment cells move perpendicularly away from the yellow pigment cells? What is the function of this chiral ‘run-and-chase’ mechanism?

To address these questions, we extend nonlocal advection-diffusion models to incorporate chiral movement, and investigate how this affects pattern formation. Using linear stability analysis and numerical simulations, we demonstrate that chirality can help to promote pattern formation, whilst simultaneously allowing ‘run-and-chase’ mechanisms to separate cell types within the pattern.

Unravelling Wrinkle Formation in a Lubricated Viscoplastic Beam

Dr Thomasina Ball

University of Warwick

Abstract:

Wrinkles or creases in the surface of a material are indicative of compression. For example, on Earth, mountain ranges formed due to the plate tectonics exhibit regular spaced folds on the surface; and buckles on the surface of ice sheets are observed due to compression from sea ice. Both examples have a layered structure with contrasting rheological behaviours where compression leads to an instability causing the stiff surface to buckle.

To model this system, often an elastic beam is considered. However, in many cases where the stiff layer is significantly broken-up, or forced to deform well beyond its yield point, other models may be more relevant, such as a substantially more viscous fluid, or a plastic material. In this talk, we consider a viscoplastic beam which is compressed while lubricated below by a thin layer of viscous fluid. The model for the beam is given by the viscoplastic analogue of the Föppl–von-Kármán plate equations for an elastic beam and connects viscous sheet models and classical theories of plastic plates.

In the limit of zero yield stress, the evolution in the linear limit is dominated by the growth of one mode. However, when a yield stress is introduced, the evolution strongly depends on how the plate yields, through either compression or bending. The yield stress also plays a significant role in increasing the minimum compressive stress required to buckle and reducing the growth rate of the instability at late-times.

This work is in collaboration with Jerome Neufeld, University of Cambridge.

Evaporation of Large Arrays of Sessile Droplets

Dr David Fairhurst

University of Edinbugh

Abstract:

Sweat on an athlete's skin, virus laden droplets in a sneeze and patterns created with an inkjet printer are all examples of multiple droplets evaporating in a shared environment. Despite the prevalence and importance of such situations, academic study has focussed almost entirely on the behaviour of individual droplets, in part due to the practical difficulties in imaging large arrays. To address this experimental challenge, we have developed and rigorously tested a 'pattern distortion' technique to allow simultaneous monitoring of the heights of many sessile droplets. We first apply the method to arrays containing hundreds of droplets, where we extract the time-dependent evaporation rate of every droplet. We find excellent agreement with three recent theories, all of which consider the droplets to only interact diffusively through the vapour phase. We then extend our experiments to arrays containing thousands of droplets. At these increased length scales, we see evidence for non-diffusive interactions which increase with Rayleigh number, a dimensionless parameter typically used to characterise the onset of convective flows. These findings have implications for many evaporation and heat-flow applications, providing a framework to predict the dynamics of the entire array.

Clustering in Predator-Prey Systems

Ms Lena Payne
Kent University

Abstract:

I will be discussing the different ways to measure cluster/pack size in a predator-prey system, and what the difference in these measures means.

Efficient Calculation of Moments in Biologically Active Taylor Dispersion Problems

Mr Nick Bryant
York University

Abstract:

In a series of papers published in the 1950s, G.I. Taylor studied the dispersion of a blob of dye transported by Poisueille flow in a tube. The equations governing these "Taylor Dispersion" problems are typically transformed into a moment analysis in order to determine the decay, drift and effective diffusion of the dye. Moments can be found by integration of the governing equations, leading to a sequence of recursively-related partial differential equations for each moment. In the case of no-flux boundary conditions, the equations can be simplified and the averaged moments found. However, this is not possible in a number if physically interesting systems such as transport through porous media. In this talk, we present a novel method of analysis that transforms the governing equations into one ordinary differential equation for a moment generating function. Moments of the distribution are then found by finding a power series solution to this equation and using the residue theorem. Further, any moment of the distribution can be found by integral operations, requiring knowledge of only the zeroth term in this power series solution. This method simplifies moment analysis in biological fluid dynamics and allows access to solutions in the presence of biologically relevant phenomena such as boundary absorption and biased motion. Studying Taylor Dispersion problems through moment generating functions may provide a more concise and insightful description of such distributions than standard recursive methods.

Models of Blood Flow in the Human Placenta

Dr Eleanor Doman

University of Manchester

Abstract:

The placenta develops alongside the fetus during pregnancy and allows for the transferal of oxygen and nutrients vital for the development of the fetus from the mother. The human placenta consists of two separate blood circulations; a fetal circulation characterised by capillaries within villous structures, and a maternal circulation characterised by porous domains surrounding the placental villi. The transport mechanisms within these regions are dependent on the flow through both circulations and the highly heterogenous tissue structure. Many adverse birth outcomes, such as those leading to stillbirth, are thought to be associated with abnormal placental tissue structure. However the interplay between tissue structure, blood flow and transport is highly complex even for healthy pregnancies.

In this talk I will discuss the unique challenges involved in understanding blood flow within the human placenta before discussing the work myself and collaborators have undertaken using (i) microfluidic experiments with suspensions of deflated PDMS capsules, and (ii) reduced-order network models. Expanding upon previous models of blood flow, I will explore how mathematical models can be used with tissue imaging data and microfluidic experiments to interpret the impact of tissue structure on blood flow. This work provides insight into the effect of tissue structure on blood flow and placental transport between the fetus and mother.

Topological fingerprints for audio identification

Dr Ximena Fernandez

University of Oxford

Abstract:

In this talk, I will present a topological approach to audio fingerprinting, utilizing persistent homology on local spectral decompositions of audio signals. Our method encodes audio content through local Betti curves, enabling accurate detection of time-aligned audio duplicates. Experimental results demonstrate superior performance in identifying identical tracks, even under obfuscations. Particularly, our approach outperforms existing methods like Shazam in scenarios involving topological distortions like time stretching and pitch shifting.

Effect of constitutive law on the erythrocyte membrane response

Dr Marianna Pepona

Durham University

Abstract:

Despite research spanning over half a century, the exact value of the shear modulus of the erythrocyte membrane is still ambiguous. This ambiguity in the shear modulus evaluation results from the constitutive law chosen to fit experimental measurements. In order to shed light on how the choice of constitutive law affects the shear modulus determination as well as the erythrocyte response, we have theoretically analysed the three most commonly employed constitutive laws, that is the Skalak, neo-Hookean and Yeoh laws, to draw relations between their corresponding shear moduli, and we have computationally investigated their effect on the response of erythrocytes subjected to optical tweezers stretching and micropipette aspiration. We have found that Yeoh law captures well the erythrocyte deformation characteristics. Most importantly, Yeoh law is able to replicate the membrane folding/wrinkling observed experimentally as opposed to Skalak law.

Persistent Diagrams and Persistent Images as statistics to constrain primordial local non-Gaussianity

Dr Gabriella Contardo

Sculo Internazionale Supiore di Studi Avanzati

Abstract:

The galaxies of our Universe form patterns in space that evolved through time, the Large Scale Structure. This cosmic web contains key information about the cosmological parameters governing our Universe. One crucial task is to construct summary statistics of the properties of the large-scale structure, that contain the relevant information to constrain cosmological parameters and that are interpretable. In this work, we explore using statistics from persistent homology to detect and characterize primordial local non-Gaussianity through the f_{NL} parameter. We investigate the ability of Persistent Diagrams (PD) and Persistent Images (derived from the PDs) computed on simulated 3D point clouds of dark matter halos to predict f_{NL}, using neural networks (Convolutional Neural Networks for the Persistent Images and Deep Sets for the PDs) compared and combined with the classical power-spectrum. Our results are encouraging and motivate the development of additional simulations covering smaller f_{NL} values and other types of primordial non-Gaussianity to perform Simulation-Based Inference in this regime ultimately.

Elementary interactions of deep-water waves in the presence of damping

Dr Raphael Stuhlmeier

Plymouth University

Abstract:

In deep water the fundamental nonlinear interaction is between quartets of waves, giving rise to the famed Benjamin-Feir (or modulational) instability. The traditional approach, dating back to the late 1960s, is to analyse this instability via temporal evolution equations, however flume experiments and some physical phenomena require a description in terms of spatial evolution. In this talk I will discuss the elementary interactions from the perspective of the spatial Zakharov equation developed by Shemer and co-workers. I will examine how non-uniform damping, such as that arising naturally in the interaction of waves with sea-ice, impacts these elementary interactions and changes the resulting dynamics.

Diffusion Geometry for Data Analysis

Mr Iolo Jones

Durham University

Abstract:

We introduce diffusion geometry as a new framework for geometric and topological data analysis. Diffusion geometry uses the theory of Markov diffusion operators to define objects from Riemannian geometry on a wide range of probability spaces. Given a sample of data, we use this theory to introduce a whole family of new geometric methods for its analysis. This includes vector fields and differential forms on the data, the wedge product, interior product, exterior derivative, Lie bracket, Levi-Civita connection, Hodge Laplacian and Riemann curvature tensor. Unlike existing methods like persistent homology and local principal component analysis, diffusion geometry is explicitly related to the existing theory of Riemannian geometry, and is significantly more robust to noise, significantly faster to compute, provides a richer topological description (like the cup product on cohomology), and is naturally vectorised for statistics and machine learning. We verify these properties on a variety of real and synthetic datasets.

Remarks on the applicability of Gardner equation to describe ISWs in a three-layer liquid system

Dr Ricardo Barros

Loughborough University

Abstract:

Recently, we have examined the conjugate states and limiting internal solitary waves (ISWs) in a three-layer liquid system and found regimes of parameters where mode-1 solitary-wave solutions with different polarities can coexist. When using weakly nonlinear theory to describe the ISWs in such physical system, it is well known that, for the special case of a symmetric configuration (Boussinesq approximation + equal density increments across the layers + same thickness of the top and bottom layers), the quadratic nonlinear coefficient of the KdV equation vanishes, although solutions of opposite polarities are predicted by the mKdV equation if the thickness of the intermediate layer is sufficiently large, in which case the cubic nonlinear coefficient is positive. This result agrees both with the strongly and fully nonlinear theories. Using strongly nonlinear theory, we investigate when mode-1 solutions with different polarities can coexist, in general, and check whether the Gardner equation (with both quadratic and cubic nonlinear coefficients) is able to make accurate predictions.

Modelling mechanical stresses in drying colloidal drops

Dr Matthew Hennessy

University of Bristol

Abstract:

The evaporation of solvent from a drop of colloidal fluid can produce a variety of intriguing fracture patterns. During drying, the liquid drop is transformed into a deformable porous gel due to the aggregation of colloids. Shrinkage of the gel leads to mechanical stresses that are released through fractures. To investigate the link between drying and fracture, we develop a model for a drying drop based on nonlinear poroelasticity in the thin-film limit. We find that the mechanical stresses are strongly controlled by the shape of the drop when it transforms into a gel. If the drop profile is parabolic, then we predict that the fractures will not align with any particular direction. However, if the drop profile is non-monotonic, then three distinct fracture patterns with preferred alignments will co-exist in the drop. We show that our theory is able to explain some of the fracture patterns seen in experiments.

Analysing travelling waves & wave packets in laterally-inhibited grids of integrate-and-fire neurons

Mr Henry Kerr

Exeter University

Abstract:

When a neuron fires, it alters the electrical activity of other connected neurons. When many neurons fire, their combined effect can create cascades of firing activity. What sort of patterns should we expect to see in these cascades? What conditions are necessary for these patterns to form?

Here, we investigate travelling waves of single or repeated firing events in both 1D and 2D grids of neurons. The neurons are modelled using an adaptation of the leaky integrate-and-fire model, with a further variant used to examine the effects of subthreshold oscillations. Connections between neurons are modelled with a Mexican Hat function, producing lateral inhibition. Neurons in close proximity excite each other through their firing, while neurons spaced at medium range will inhibit one another's activity.

We present a construction of these waves in a continuum model, and stability results for specific perturbations. We also present GPU code to support simulation of a broad class of neuron models, including the use of timestep-free methods where possible.

PDE modelling and simulation of intracellular signalling pathways

Ms Sofie Verhees

Heriot-Watt University

Abstract:

Communication and interactions between cells happen mostly through intercellular signalling processes. These signalling pathways are important in all physiological activities of the cell, such as cell division, cell movement, immune response, and tissue development. In many of these signalling pathways, the chemical processes and mechanics of the cell work together. However, how exactly these two phenomena communicate is not well known. A common way to model the chemical processes of cell signalling pathways are reaction-diffusion equations. The mechanical properties of the cell are modelled assuming elastic constitutive relationships. Regarding the chemical process, our model includes the diffusion of signalling molecules and membrane receptors, and the reactions between the molecules and receptors. This is coupled to the mechanical properties such that the mechanics of the extracellular matrix influences the interaction between the signalling molecules and the results of the signalling pathways affect the deformation of the cell. To explore this coupling, we model the cell signalling processes involving the Rho signalling molecule, which is known to interact with the mechanical properties of the cell and the extracellular matrix. Simulation results, benchmarking and a comparison to experimental observations will also be presented.

An asymptotic framework for model differentiation and comparison in flood risk estimation and rainfall-runoff modelling

Dr Phil Trinh

Bath University

Abstract:

One of the key problems in flood estimation is to predict the flow in the river after an intensive rainfall. Hydrologists use a variety of methods for such predictions, ranging from physical (PDEs) models to conceptual models to statistical models. These models are often based on different assumptions; such assumptions may be subtle and hidden, and perhaps contradictory. The work of Peel & McMahon (2020) catalogued nearly 300 rainfall runoff models used by researchers. There has been significant criticism in the excessive number of models and their unknown differences.

During this talk, I will discuss the power of asymptotic analysis in providing tools to systematically differentiate between models; this serves as both a methodology and philosophy for understanding the limitations and relationships between disparate models. I will then describe our work in developing simple but rigorous benchmark scenarios for coupled surface-subsurface flows, and what we can learn from the approach.

Optical skyrmions

Dr Jörg Götte

University of Glasgow

Abstract:

Skyrmions are topological states in matter for which the spin and orbital degree of freedom are inseparably linked. We show how such complex states can be created in paraxial optical beam and how such optical skyrmions inherit the mathematical properties of matter skyrmions. Realising skyrmions and other generalised vortices in optics allows us to study the propagation of topological defects and to identify skyrmion field lines as a skeleton for variegated optical polarisation structures.

Self-organization in motile signaling bacteria

Mr Wesley Ridgway

University of Oxford

Abstract:

Chemical signalling, or quorum sensing (QS), promotes a variety of behaviours in bacteria, from biofilm formation to swarming. By engineering QS systems to couple with genes controlling motility in bacteria, tunable spatio-temporal patterns can be generated in vitro. However, it is not well-understood how the gene-regulatory networks in individual cells affect emergent population-level patterns.

In this talk, we investigate the role of the gene-regulatory network on self-organization in populations of motile bacteria that interact via QS. We use a multiscale, chemically structured continuum model, which we obtain by upscaling a microscopic agent-based model via the Fokker-Planck equation. Importantly, our modelling framework retains the full distribution of internal genetic states through chemical structuring, rather than replacing it with a mean-field. We use a novel WKBJ-like asymptotic framework to obtain analytical criteria for the onset of several types of emergent behaviour and compare our findings with conventional reaction-diffusion models.

Embedding Soft Synergies into Soft Materials for Intrinsic Compliant Robotic Hand Grasping

Dr Emanuel Nunez Sardinha

University of the West of England

Abstract:

Replicating the grasping capabilities of human hands is one of the greatest challenges for control in robotics. Imitating human grasping synergies -joint configurations, movements and relationships for the hand posture during grasping tasks - can significantly simplify control. However, achieving this via direct control can result in very complex software or hardware systems, especially for underactuated designs. Embedding directly into the hardware can simplify control drastically, but such robotic hands can be complex, costly, and difficult to design. Here we show an approach to embed and tune grasping synergies directly onto simple soft silicone structures, producing compliant anthropomorphic fingers with human-like motion. We follow a human-in-the-loop optimisation approach, modifying key control variables in the finger structure to reach the individual target movement ratios. A validated simulation is used as a predictive design tool from which joint behaviour is defined based on the data for the first human grasp synergy. Binary search is used to minimise error from the target value. The result is a low-cost compliant hand capable of replicating a programmed human grasp synergy using a few mechanical components, actuated by a singular DOF input. Grasping quality was demonstrated with a variety of objects from the YCB grasping dataset, with a success rate of 80% for the test items and grip strength comparable to similar compliant hands. By embedding the main human grasping synergy to individual soft silicone fingers, we can produce a functional under-actuated hand with fingers composed uniquely of soft materials and no electronic parts.

Higher order far-field boundary conditions for crystal defects computations

Dr Julian Braun

Heriot-Watt University

Abstract:

Many properties of crystalline solids are heavily affected by lattice defects on the atomistic scale. To understand and compute these defect we consider atomistic models capturing both the defect core and the long-range elastic fields the defect creates. The structure of the long-range elastic field for a defect equilibrium is revealed by a rigorous far-field expansion, showing that it can be expressed as a sum of continuum correctors and discrete multipole terms both of which are computable. We develop a novel family of numerical schemes that exploit the multipole expansions to accelerate the simulation of crystalline defects. To enclose the simulation in a finite domain, a theoretically justified approximation of elastic multipole tensors is introduced, which leads to a novel moment iteration resulting in higher order boundary conditions. Prototypical numerical examples of point defects and dislocations are considered to show that our proposed numerical scheme matches the accelerated convergence rates in terms of computational cell size given by the rigorous convergence estimates.

Turing patterns on the move

Mr Vit Piskovsky

University of Oxford

Abstract:

Spatial patterns are ubiquitous in nature, from chemical reactions on the molecular scale to zebrafish pigmentation on the cellular scale and clustering of vegetation in arid environments on the organismic scale. Surprisingly, a single mathematical mechanism, known as the Turing mechanism, is thought to underlie the formation of all these patterns. Due to its universality, the Turing mechanism has been hypothesised to explain the patterns in the spatial distribution of motile biological species, namely predators and prey. Yet, despite these predictions, such patterns have not been empirically tested or observed. In the talk, I will suggest an explanation for this lack of empirical evidence, proving that the evolution of organismic motility destabilises Turing patterns. While the evolution of predators and prey kills organismic Turing patterns, such patterns can play an important role at other scales of animal movement. In the second part of the talk, I will suggest how Turing patterns might emerge in the brain activity of moving animals and how such patterns can positively contribute to their fitness.

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Model based control of soft robots

Dr Cosimo Della Santina

Delft University of Technology

Abstract:

Soft robots have continuously deformable bodies made to resemble invertebrate animals like octopuses. This makes modeling and controlling them hard. In the past decade, substantial progress has been made in developing a low-level artificial brain for soft robotic systems that can make them execute precise motions and eventually exploit the intelligence embedded in their complex mechanical structures. In this talk, I will briefly introduce this grand challenge within the soft robotic field and then present the activities of my groups towards a model-based view of its solution. I will show how models can be combined with nonlinear control theory leading to precise and dynamic task execution.

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Spatial models of forest-savanna bistability

Dr Denis Patterson

Durham University

Abstract:

Empirical studies suggest that for vast tracts of land in the tropics, closed-canopy forests and savanna are alternative stable states, a proposition with far-reaching implications in the context of ongoing climate change. Consequently, numerous mathematical models, both spatially implicit and explicit, have been proposed to capture the mechanistic basis of this bistability and quantify the stability of these ecosystems. We present analysis of a spatially extended version of the so-called Staver-Levin model of forest-savanna dynamics (a system of nonlinear partial integro-differential equations). On a homogeneous domain, we uncover various types of pattern-forming bifurcations in the presence of resource limitation, which we study as a function of the resource constraints and length scales in the problem. On larger (continental) spatial scales, heterogeneity plays a significant role in determining observed vegetative cover. Incorporating domain heterogeneity leads to interesting phenomena such as front-pinning, complex waves, and extensive multistability, which we investigate analytically and numerically.

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Exploring the role of EMT and Cell Motility in Ovarian Cancer Progression

Mr Samuel Oliver

Swansea University

Abstract:

Epithelial-to-mesenchymal transition (EMT) plays a key role in the progression of cancer tumours, making treatment significantly less successful for patients. EMT is when a cell changes phenotype and possesses different behaviours to those previously exhibited such as enhanced drug resistance, higher cell plasticity, and increased cell motility. This enhanced cell movement increases the metastatic ability of a tumour. Metastatic cancers are responsible for 90% of all cancer related deaths. As a result, it has become essential to encapsulate this change and quantify this impact. Here, we use a multiscale mathematical framework for two cell lines, OVCAR-3 and SKOV-3, to investigate EMT and the role of the transition over time. The model is a 3D agent-based multiscale model built upon a Physicell framework, incorporating the experimentally observed cellular changes and the effects of microenvironment. OVCAR-3 and SKOV-3 tumours possess highly contrasting tumour layouts, meaning we have a range of different settings to test and study. Having such an adaptable model allows a variety of results to be obtained faster with fewer changes needed to the core dynamics, enabling comprehensive in silico analysis. The model encapsulates the biological observations and trends seen in tumour growth and development, reassuring the fact it is a viable model from which to make predictions and insights. The study highlights the key processes in the multistage EMT, with sensitivity analysis showing the input parameters with the biggest effect on the outcome of the tumour over time.

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Discrete differential geometry-based model for the snapping analysis of axisymmetric shells

Dr Weicheng Huang

Newcastle University

Abstract:

Axisymmetric shells can be found in numerous scenarios in both the natural environments and engineering applications, thus an efficient model would be crucial for the comprehensive understanding on their mechanical behaviors. In this work, we propose a novel discrete model for the nonlinear buckling and snapping analysis of axisymmetric shells based on differential geometry theory. Based on discrete differential geometry, the axisymmetric shell is discretized into interconnected one-dimensional elements along the longitudinal direction, and the in-plane stretching and out-of-bending potentials of shell mid-surface are formulated based on the geometric principles, and, therefore, the model can naturally incorporate geometric nonlinearities associated with large deflections and rotations. We first verify the developed discrete model through the quantitative comparison with the theoretical models and finite element simulations reported in literature. Next, with the developed model, we systematically investigate the bistability and the snap-through inversion of a spherical shell under indentation, in which the nonlinear boundary contact is involved. A simple geometric model is later built to reveal the contact mechanism between the flexible shell and the rigid indenter. In addition, the magnetic-induced snapping of axisymmetric shell is also systematically investigated by incorporating our discrete numerical model with magneto-elastic constitutive relation.

What TDA can tell us about the nature of matter

Dr Jeff Giansiracusa

Durham University

Abstract:

Physics describes the world in terms of quantum field theories; the inside of protons and neutrons is described by one particular theory called QCD, which says that things are made out of particles called quarks and gluons. It's a very successful theory, except for one annoying detail: nobody has ever seen a quark or gluon! This paradox is explained away by a hypothetical mechanism called 'confinement' that nobody really understands. We see clear evidence of confinement in experiments, and we can watch it happen in Monte Carlo simulations on supercomputers. These simulations produce vast amounts of data, and so understanding confinement should be a data science problem. I'll explain how persistent homology might just be the tool we need for this problem.

Motility Induced Phase Separation in Quorum Sensing particles

Dr Devi Prasad Panigrahi

University College London

Abstract:

Motility induced phase separation (MIPS) has attracted a lot of attention from the biophysics community because of its potential application in predicting the formation of bacterial biofilms. It is also known that bacteria can communicate with each other using chemical signals, by a process known as Quorum sensing. While the dynamics of phase transition of self-propelled particles has been studied in detail, very little is known about the interplay between MIPS, and Quorum sensing. Given the fact that Quorum sensing is ubiquitous in bacterial colonies, it is very important to consider its effects while studying bacterial aggregation. Our research is the first to show that a simple model for Quorum sensing, coupled with a gene-regulatory network can give rise to MIPS, even in absence of self-propulsion. We also present a systematic study on how this fundamental mechanism is altered by the presence of volume exclusion and self-propulsion.

Marangoni-enhanced spreading of alcohol droplets drying on a solid

Dr Lisong Yang

Durham University

Abstract:

The drying of liquids is a complex process that can hide a wealth of dynamics occurring inside the liquid. The fundamental understanding of the process lends us a key to changing the distribution of the deposit, which is of importance in spraying agriculture, spraying coating, inkjet printing and fabrication of (opto)electrical and biological devices. We present two pL-droplet drying cases on a solid under Marangoni-enhanced spreading (ref1-4): 1) isopropanol alcohol (IPA) droplet containing a small amount of a second alcohol, such as 2-butanol, and 2) IPA droplet under controlled relative humidity. In the first case, enhanced evaporation of the more volatile solvent with lower surface tension (IPA) at the contact line leads to outward Marangoni flow from apex to contact line, which results in the enhancement of the spreading and delay of the contact line pinning; In the second case, as condensation is fastest at the contact line and water raises the surface tension of alcohol at the contact line, there is again an outward Marangoni flow from apex to the contact line. Marangoni flows are strong enough in both cases to significantly deform the drop, forming submicron-thin pancake-like shapes under certain conditions. We demonstrated that the formation of pancake-like shapes can mitigate the coffee-ring effect in given IPA blends containing nanoparticles. We developed a quantitative model that captures the essential physics of the problems and shows good agreement with experiments in both cases. The model provides a predictive capability to modulate the shape of tiny droplets with implications in inkjet printing and film fabrication.

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1. D. Pesach and A. Marmur, Langmuir, 3, 519-524 (1987)
2. G. Hu et al, Science Advances, 6, eaba5029 (2020)
3. A. A. Pahlavan et al, Phys. Rev. Lett. 127, 024501 (2021)
4. L. Yang et al, PNAS, 120, e2302653120 (2023)

Matched asymptotic expansions for the rocking can and other unusual problems

Dr Cameron Hall

University of Bristol

Abstract:

The method of matched asymptotic expansions (MAEs) is a valuable analytical tool in applied mathematics that is often associated with “boundary layer” problems characterised by a small parameter multiplying the highest derivative in a boundary value problem. However, MAEs are not limited to these problems; as described in Hinch’s “Perturbation Methods” and elsewhere, MAEs can be used wherever there is a change of dominant balance between different regions of the solution domain, regardless of whether there is a small parameter multiplying the highest derivative.

In this talk, I will describe a simple ODE problem based on the dynamics of a rocking can [1] in which MAEs can give valuable insights into the solution but where there is no small parameter multiplying the highest derivative. This will be used to illustrate the value of MAEs in problems from nonlinear dynamics, and to introduce other problems that can be used as examples and exercises that illustrate the fact that MAEs are not limited to situations with a small parameter multiplying the highest derivative.

[1] Collins, B. W., Hall, C. L., and Hogan, S. J. (2023) SIAM JAppDynSys 22(4):3358-3389, DOI:10.1137/23M1551031.

Exploiting the non-linear dynamics of dielectric elastomers for soft robotics

Dr Andrew Conn

University of Bristol

Abstract:

Dielectric elastomers (DEs) are compliant electromechanical transducers with high energy densities that can generate large actuation strains of over 50%. They offer great potential as lightweight actuators in soft robotic systems. They have recently been demonstrated for new types of soft robotic pumps and grippers but their non-linear dynamics and hyperelastic characteristics make their design and control challenging. In this talk, it will be shown how electromechanical modelling using hyperelastic strain energy functions can be used to capture the non-linear dynamic response of coupled DE membranes. The unique dynamic characteristics of different membrane coupling mechanisms, such as encapsulated fluid and magnetic force, will be analysed. The modelling approach will be validated against experimental results and it will be shown how the electromechanical instabilities inherent to DEs can be prevented for robust performance.

Spatial Modelling of Blood Vessel Remodelling in the Tumour Microenvironment

Mr Nicholas Fan

Oxford University

Abstract:

The tumour microenvironment is a complex system with many interacting cell populations which support or attack tumour cells. Adding to the complexity, altered behaviour of fibroblasts, a cell type responsible for producing extracellular matrix (ECM), creates a dense network of ECM which inhibits mobility of tumour fighting immune cells. These modified fibroblasts also assist the tumour in remodelling surrounding blood vessels, causing chaotic blood flow and hypoxia.

The aim of our research is to understand how vessel remodelling impacts the spatial localisation of cells in the tumour microenvironment, and the effect this has on a tumour's survival and sensitivity to treatment. We approach this problem by creating an off-lattice agent-based model of a tumour, focusing on mechanisms of vessel and matrix remodelling. Our model builds upon a tumour spheroid simulation, adding simplified mechanisms of vessel and matrix remodelling to create a minimal description of the tumour microenvironment. Building our model from this foundation makes it modular and flexible, allowing us to analyse the individual effect of each mechanism. The individual cell resolution from agent-based modelling is particularly suited to study the spatial localisation of cells in the tumour. Immunofluorescence and immunohistochemistry stained microscopy images are of similar resolution to our model, allowing us to validate our model against real tumours.

Key questions we address with our model include investigating how vessel remodelling changes a tumour's sensitivity to treatment, especially radiotherapy and immune control. Hypoxic regions created when vessels become occluded cause tumour quiescence which increases resistance to radiotherapy, and also induces processes which increase cancer cell motility, both of which result in poor patient prognosis. In future work we will incorporate T-cells, immune cells which directly kill tumour cells, into our simulation. We will use our model to investigate the effectiveness of the immune system in controlling tumours, focusing on the effects of T-cell exhaustion, a process in which T-cells become inactivated within the tumour microenvironment.

Mathematical insights into context-specific Notch signalling: endothelial cells and intestinal crypt niche

Dr Daria Stepanova

Centre de Recerca Matemàtica

Abstract:

The Delta-Notch pathway serves as an evolutionarily conserved mechanism facilitating contact-dependent cellular communication. This signalling cascade is initiated when a Notch receptor on one cell binds to the transmembrane ligand Delta on an adjacent cell. The classical pattern emerging from this signalling pathway is characterised by cells exhibiting alternating gene expression signatures (phenotypes), often described as a distinctive salt-and-pepper pattern. However, Notch signalling in biological systems can display remarkable versatility, exhibiting variations in cell spacing and positioning to fulfil tissue-specific functions. In this talk, we will consider two particular cases: (i) the patterns emerging during the growth of blood vessels in endothelial cells, orchestrated by the interplay of Notch signalling and Vascular Endothelial Growth Factor (VEGF) activation, and (ii) the patterns observed within the intestinal crypt niche, where Notch signalling is coupled with the Wnt pathway. We will discuss the mathematical formulations of models for each case, shedding light on their potential to enhance agent-based models. The emphasis lies on providing a more accurate representation of cell function influenced by the local microenvironment, defined by its signalling cues rather than constrained by predetermined rules.

Pattern formation in active solids

Dr Anton Souslov

Cambridge University

Abstract:

Active solids consume energy to allow for actuation and shape change not possible in equilibrium. I will focus on the elasticity of systems as wide-ranging as living matter, nanoparticles, and mechanical structures composed of active robotic components. I will discuss how in lattices of robots, inertia and active elasticity conspire and give rise to new varieties of pattern formation. These results provide a theoretical underpinning for recent experiments and point to the design of novel soft machines.

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The curious case of the chattering chalk

Professor Alan Champneys

University of Bristol

Abstract:

This talk provides an update to work over a number of years on the so-called Painleve paradox in mechanics which is thought to underlie what happens when you try to push (rather than drag) a piece of chalk across a blackboard. This paradox involves the fundamental incompatibility between models of dry friction and rigid contact. Under certain circumstances, open sets of initiation conditions of rod-like objects in frictional unilateral contact can reach a point of impasse, a finite time singularity in which the normal contact stiffness effectively becomes negative. Previous work using asymptotic analysis suggested a dichotomy between impact and lift-off at this singularity. We extend that analysis to show that dichotomy is unimportant, as there is a new timescale under which impact is always completed, so that velocity changes by an O(1) amount in infinitesimal time. Some continuations from that point still contain a point of indeterminacy called reverse chatter, which is like watching a bouncing ball coming to rest in reverse time. Whether this explains the juddering motion of chalk remains an open question.

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Overview of Mathematics and Control Theory in Soft Robotics

Mr Vijay Chandiramani

Bristol Robotics Laboratory (joint EPSRC, UoB, UWE)

Abstract:

Soft robotics, a fascinating field, is multidisciplinary in nature encompassing material science, biology, mathematics, continuum mechanics, and robotics, as well as distinct sub-fields within each of these areas. The challenge of controlling soft robots is particularly interesting because it combines fundamental questions with practical applications. Soft robots are inherently under-actuated and nonlinear mechanical systems and it is the challenge of soft roboticists to harness these effects to generate precise motions with the few actuation sources available and execute controlled interactions with an external environment or store energy during dynamic movements.

This workshop aims to present various researchers using who are using applied mathematics, control theory and machine learning to address the soft robotics control challenge using model-free and model-based approaches.

Superfluid Vortices in Four Spatial Dimensions

Professor Hannah Price

University of Birmingham

Abstract:

Quantum vortices in superfluids have been an important research area for many decades. Naturally, research on this topic has focused on two-dimensional (2D) and 3D superfluids, in which vortex cores form points and lines, respectively. Very recently, however, there has been growing interest in the quantum simulation of systems with four spatial dimensions; this raises the question of how vortices would behave in a higher-dimensional superfluid. In this talk, I will introduce our recent works looking at vortices in 4D superfluids under rotation, where the vortex core can form a plane. By solving the Gross-Pitaevskii equation, we find that suitable rotations can stabilise a pair of vortex planes intersecting at a point. We also find states containing pairs of skewed and curved vortex planes, which have no direct analogue in lower dimensions. Our work opens up many directions for future research into the phenomenology of topological defects in higher dimensions.

Sexual behaviour, mobility, transmission clusters: modelling a discrepancy between the epidemic of HIV in Sub-Saharan Africa and Europe

Dr Francesco Di Lauro

University of Oxford

Abstract:

Reductions in HIV infection rates in sub-Saharan Africa (SSA) do not fully reflect the increases in viral load suppression achieved through universal testing and treatment (UTT). Heterogeneities in risk and treatment uptake contribute to this disparity. In North America and Europe, HIV transmission is concentrated among key populations with frequent super spreading. The extent to which African HIV epidemics are similarly linked to key populations is poorly understood. Molecular epidemiology can shed light on these dynamics.

We analyzed phylogenetic clustering patterns using HIV-TRACE on HIV sequences from the PANGEA-HIV consortium. Clustering was compared across sites and investigated with agent-based simulations (PopART-IBM and EMOD-HIV). We also compared clustering between men who have sex with men (MSM) in the UK and Netherlands and heterosexual populations in SSA.

The distribution of clusters in SSA was more homonegenous and typically centred around a smaller average than in MSM populations in Europe, even when accounting for sampling fraction and epidemic phase. European MSM clusters are characterised by large overdispersion.

A simple branching process model shows that phylogenetic clustering is sensitive to the skewness of the transmission offspring distribution.The extent to which the dissimilarity in offspring distributions between MSM in Europe and heterosexual in SSA can be attributed to the degree distribution of sexual networks is investigated with a simple sexual network model.

Our findings align with agent-based simulations in which HIV transmission originates from the general population without necessarily significant contributions from key populations, with important implications for the future of HIV prevention in SSA.

Breaking the (Mathematician) Mould

Miss Robyn Goldsmith

Lancaster University

Abstract:

Many of us hold a preconceived idea of the "typical" mathematician. From a young age, I knew I didn't fit this image. Now, as a PhD student, I aim to increase visibility of women in mathematics and excite non-scientific audiences through science writing, outreach talks and social media. I will discuss my experiences of how maths communication can help challenge preconceptions, remove barriers to belonging and diversify the view of what it means to be a mathematician.

An Application of Topological Data Analysis to Drift Wave Turbulence

Mr Sage Stanish

University of Glasgow

Abstract:

Topological Data Analysis (TDA) is a field that provides a set of tools to compute the topology of arbitrary data. The computed topology quantitatively identifies features in complex data. Recently, TDA has found some application to classical fluid problems like Kolomogorov flow and Rayleigh-Bénard convection where patterns in the flow, like the onset of convection, have a distinct topological signature. This talk provides an introduction to TDA via the application of Persistent Homology to Magneto-hydrodynamic turbulence. We consider the Hasegawa-Wakatani model for drift-wave turbulence and show that TDA describes the turbulent to zonal flow transition better than standard methods.

Quantitative analysis of the drivers and consequences of the spatial pattern of cancer invasion

Dr Robert Jenkins

The Francis Crick Institute

Abstract:

For patients with cancer, the spatial pattern of the tumour tissue has significant prognostic value. We neither fully understand the mechanisms that drive the formation of tumour pattern or the impact of these patterns on patient outcome. Here we use a combination of quantitative analysis, in-vitro and in-vivo experimentation and clinical patient data to help answer these questions. Tumour tissue does not develop in isolation and its interaction with adjacent tissue influences its development. We pay particular attention to how the stroma, including extracellular matrix (ECM) that supports tissue structure, and fibroblasts that maintain this ECM, can influence and predict outcome. There are two strands to our research. Firstly, we use mathematical models to investigate cellular mechanisms that drive differing spatial patterns of tumour cell invasion and ECM architecture. Secondly, we use quantitative methods to describe the spatial patterns in tumour tissue including lung and prostate, focussing in particular on the ECM architecture rather than simply the cancer-cell architecture. In the case of prostate, we demonstrate how this can be used to predict likelihood of recurrence of disease following curative therapy.

Magnetic Field Topology in Turbulent Magnetic Reconnection

Dr Alexander Russell

University of St Andrews

Abstract:

Magnetic reconnection is a fundamental plasma process that rapidly converts magnetic energy into other forms. It is responsible for many impulsive phenomena, including solar flares and disruptions in fusion experiments. Turbulence has long been believed to play a major role in magnetic reconnection, and we have recently entered an exciting era in which state-of-the-art simulations generate turbulence inside the layer where reconnection occurs, enabling new progress on the interplay between turbulence and reconnection. While theory is still catching up with new-generation simulations, it is already clear that the magnetic field topology inside the turbulent reconnection layer is of prime importance. This talk provides an introduction to turbulent magnetic reconnection and the new-generation of simulations, then discusses the role of magnetic topology, including: local coherence of magnetic field line bundles in MHD turbulence, conservation of magnetic helicity, and implications for the theory of turbulent reconnection.

Dynamics of pancake-like geophysical vortices: from waves to (bulk) turbulence?

Dr Jeremie Vidal

University Grenoble Alpes

Abstract:

Rotating stratified flows often exhibit (almost) isolated pancake-like vortices, whose lifetime may depend on small-scale turbulence. Here, we present a reduced model to describe the bulk dynamics of such vortices (e.g. Mediterranean eddies or Jupiter's vortices). We consider a fluid enclosed within a rotating triaxial ellipsoid, which is stratified in density with a constant Brunt-Väisälä frequency (using the Boussinesq approximation). We first investigate the linear wave motions in such a model, which are governed by a mixed hyperbolic-elliptic equation for the velocity. As in the rotating non-stratified case [1], we prove that the wave spectrum is pure point in ellipsoids (i.e. only consists of eigenvalues) with smooth polynomial eigenvectors [2]. Then, by combining microlocal analysis and numerical computations (obtained with a bespoke Galerkin method), we uncover the existence of low-frequency waves below the usual cutoff frequency of inertia-gravity waves [2,3]. Finally, we explore whether the elliptical instability could sustain wave turbulence in such stratified vortices.

Joint work with Prof. Y. Colin de Verdière, funded by ERC (grant agreement #847433).

[1] Colin de Verdière Y. & Vidal J., 2023, The spectrum of the Poincaré operator in an ellipsoid, Submitted (arXiv:2305.01369)

[2] Vidal J. & Colin de Verdière Y., 2024, Inertia-gravity waves in geophysical vortices, Accepted in Proc. R. Soc. A

[3] Colin de Verdière Y. & Vidal J., 2024, On gravito-inertial surface waves, Submitted

Higher-Dimensional Methods in Cellular Haptotaxis Applied to Cancer

Dr Arran Hodgkinson

Queen's University, Belfast

Abstract:

Oncological haptotaxis is well-established as a dynamic process, where in cell migration affects the underlying extra-cellular matrix (ECM) and, conversely, the structure of the ECM will influence migratory outcomes. Moreover, the density of the ECM and collagen substrate is known to play a significant role in this process. We construct a higher-dimensional, partial differential equation (PDE) model, which allows one to concurrently explore both the cellular reorganisation of the surrounding tissue and extra-cellular matrix (ECM) and the resultant effect on directed cellular migration. Careful choice of numerical techniques and parameterisation approaches are required in order to generate solutions to these highly dynamic PDEs. Exploratory simulations were generated to explore the effects of these dynamical quantities upon the invasive capacity of simulated cancer cells, wherein several physical measures of behaviour may be quantified within the framework. We find an ability for the model to recapitulate important results from the literature, while significant additional developments are made possible by the novel modelling approach to haptotaxis.

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Diffraction by a set of collinear cracks on a square lattice: an iterative Wiener-Hopf method approach

Ms Elena Medvedeva

University of Manchester

Abstract:

The diffraction of a time-harmonic plane wave on collinear finite defects in a square lattice is studied. This problem is reduced to a matrix Wiener-Hopf equation. This work aims to adapt the recently developed iterative Wiener-Hopf method to this situation. The method was motivated by wave scattering in continuous media but it is shown here that it can also be employed in a discrete lattice setting. The numerical results are validated against a different method using discrete Green's functions. Unlike the latter approach, the complexity of the present algorithm is shown to be independent of the length of the cracks.

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Relaxation of Magnetic and Vortex Braids: A Comparison

Professor Gunnar Hornig

Dundee University

Abstract:

In hydrodynamics and magnetohydrodynamics, we observe that at sufficiently high (magnetic) Reynolds numbers, the dynamics consist of a mixture of ideal behaviour, where the topology of a vortex or magnetic field lines is conserved, with localised events in the form of a vortex or magnetic reconnection, where the topology changes. High Reynolds numbers also imply that the reconnection events typically occur in cascades or avalanches and lead to a lower energy state that can have a significantly different topology.

We compare simulations of the relaxation of magnetic and vortex braids, which show some surprising similarities even though there are significant differences in the underlying equations. In both cases, the topology simplifies similarly despite the evolution of helicity, energy or enstrophy being very different. We will analyse the reasons behind these similarities and differences.

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Inverse design for the controlled boundary shape kirigami tessellation

Mr Xiaoyuan Ying

University of Edinburgh

Abstract:

The proposed research endeavors to bridge traditional Japanese artistry kirigami with future engineering applications. Kirigami, an intricate paper-cutting technique, offers more than aesthetic and decorative qualities; it serves as an insight into engineering innovations. Previous works have primarily focused on either the inverse design of purely kinematic, generalized kirigami tessellation or the mechanics of square rotating tessellation. However, the challenge remains of creating structures that can morph from one compact configuration to a desired shape with a given mechanical response by incorporating kirigami tessellation, topology, kinematics, and mechanics. This research conducted a comprehensive investigation into the kinematics and mechanics of kirigami and proposed an inverse design strategy for achieving tuneable and predetermined boundary shapes within a generalized kirigami framework. The main objective is to formulate an optimization framework and adopt kinematic-mechanical principles in a two-stage process. The initial stage involves the development of an algorithm for the optimization of cut patterns that can morph into specific shapes under prescribed stimuli. Subsequently, the second stage is the validation of the optimized cut patterns through finite element analysis and experimental tests. Furthermore, we derived a modified energy-based model using FEA results and experimental results for future engineering applications.

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Relative magnetic helicity in multiply connected domains

Dr David MacTaggart

University of Glasgow

Abstract:

Relative magnetic helicity is an important measure of a magnetic field's complexity, with many applications in both astrophysics and laboratory plasmas. In this talk, I will show how to define relative magnetic helicity in very general multiply connected domains. In particular, I will reveal the precise roles of both physics and topology in defining this general form of relative helicity, from which the helicity expressions commonly found in the literature can be derived.

Heterogeneous structuring in traveling wave solutions of a trait-structured Keller-Segel model

Ms Viktoria Freingruber

University of Edinburgh

Abstract:

In this study we generalise the Keller-Segel (KS) model, a well-established framework used for investigating chemotaxis-driven invasion across diverse biological contexts. We extend the model to incorporate intrinsic heterogeneity within the cell population, represented by the percentage of occupied membrane receptors by ligands. This trait is subject to change through ligand attachment/detachment processes. The ligand serves a dual role as a chemoattractant and, in specific scenarios, a nutrient for cell proliferation. The resulting framework consists of a non-local partial differential equation governing the dynamics of the cell population, coupled with a diffusion-reaction equation capturing the evolution of the ligand. Employing a Cole-Hopf transformation, we formally derive properties of traveling wave solutions, offering analytical insights into the system's behavior. We conduct numerical simulations that align closely with our analytical results, thereby enhancing the robustness and applicability of the extended KS model. This work contributes valuable perspectives to the understanding of chemotaxis-driven invasion in heterogeneous cell populations and underscores the significance of trait-structures in modeling complex biological phenomena.

Axion String Source Modelling

Dr Amelia Drew

University of Cambridge

Abstract:

Cosmic strings, sometimes referred to as vortices, are a class of topological defect that are predicted to form in the early Universe in certain beyond-the-Standard-Model theoretical physics models. If detected, cosmic strings would constitute direct evidence of new physics and potentially provide a direct window into the nature of dark matter. In this talk, we discuss our numerical results from adaptive mesh refinement simulations of standing-wave and travelling-wave configurations of axion strings, a specific class of cosmic string whose energy density depends on the ratio between the string thickness and string separation in the context of a cosmological network. We show that our simulations agree with analytic models of axion string radiation in the thin-string limit, and present models for the dependence of the massive and massless radiation emitted from the strings on parameters including the string width and curvature. We place this work in the context of dark matter searches and gravitational wave predictions for cosmic strings.

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A Generelised Hydrodynamics approach to the Boussinesq equation: a prototypical example of 2D stationary soliton gas

Dr Thibault Bonnemain

King's College London

Abstract:

Generalised hydrodynamics (GHD) is a recent and powerful framework to study many-body integral systems, quantum or classical, out of equilibrium. It has been applied to several models, from delate Bose gas to the XXZ spin chain, the KdV soliton gas and many more... Yet it has only been applied to (1+1)D systems and generalisation to higher dimensions of space is non-trivial. We then turn to the Boussinesq equation which, while generally considered to be less physically relevant than the KdV equation, is interesting as a stationary reduction of the (boosted) KP equation, a prototypical and universal example of a nonlinear intergrable PDE in (2+1) dimensions. I will propose a heuristic approach inspired by the Thermodynamic Bethe Ansatz in order to construct the GHD of Boussinesq soliton gas; the main motivation being that this is to be seen as a first step in the construction of the KP soliton gas, yielding insight on some classes of solutions from which we may be able to build an intuition on how to devise a more general theory. Such approach allows for a statistical mechanics interpretation of the Boussinesq soliton gas that comes naturally with the GHD picture. This also offers another perspective on the construction of anisotropic bidirectional soliton gases previously introduced phenomenologically by Congy et al.

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Instability of wall-bound filaments induced by molecular motors causes rotational cytoplasmic streaming

Dr Debasish Das

Strathclyde University

Abstract:

Cytoplasmic streaming is the flow of the cytoplasm inside the cell, driven by forces from the cytoskeleton. It is usually observed in large plant and animal cells. In this work, we are concerned with the latter and focus on the oocytes (egg-cells) of Drosophila melanogaster fruit flies (common fruit-flies) that produce different cytoplasmic flows as they develop. During a specific stage of maturation, the microtubule cytoskeleton reorganizes to form a layer of cortically bound microtubules around the periphery of the oocyte. The transport of organelles along these microtubules creates a rotating flow of the oocyte cytoplasm, known as ooplasmic streaming. This streaming flow enables mixing of the cytoplasm transported from the nurse cells with the pre-existing oocyte cytoplasm, as well as possibly facilitating the localization of certain organelles within the oocyte.

Our work sheds new light on the origins of this cell-spanning rotational flow of the cytoplasm using solid and fluid mechanics. We model the microtubule as a flexible filament, anchored on a rigid body and immersed in a Stokes fluid. The molecular motor creates the necessary biological activity in the system by applying a point force on the tip of the microtubule. By investigating the behaviour of the resulting fluid-structure interaction problem, we reveal a novel elasto-hydrodynamic instability that causes the microtubules to align with the cell walls, generating rotational flows.

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Spatio-temporal dynamics of nutrient exchanges in microbial active matter

Dr Praneet Prakash

University of Cambridge

Abstract:

Microorganisms inhabit highly fluctuating environments and survive in a low-nutrient resource bath. It is now well recognized that symbiotic relationships between microbes play a vital role in their survival. The existence of such interaction raises general questions about the spatio-temporal dynamics of nutrient exchanges. Here we experimentally and theoretically examine a model system of this problem – bacteria, an obligate microbe capable of chemotactic response towards oxygen, in a co-culture with green algae, which produce oxygen when illuminated. Even in their simplest arrangement in a localized illuminated domain, we find a complex dynamics involving nutrient exchanges, enhanced algal diffusivity due to the bacteria, and a stochastic version of "flux expulsion".

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Bifurcations and nonlinear dynamics of the follower force model for active filaments

Ms Bethany Clarke

Imperial College, London

Abstract:

Slender filaments such as microtubules are ubiquitous in nature, driving fluid flow at the microscopic scale. Molecular motors like dynein and kinesin translocate along microtubules, causing a range of both steady and time-dependent behaviours. The coordinated motion of microtubules can lead to phenomena like cytoplasmic streaming and ciliary beating, generating fluid flows on larger scales. In this talk we provide a comprehensive overview of the emerging dynamics of the most fundamental model that captures the effect of molecular motors on a single filament; the follower force model, whereby a compressive force is imposed at the filament tip. We vary both the strength of this force and the slenderness of the filament to explore the resulting state space. Employing a Jacobian-Free Newton-Krylov method, we establish both steady and time-periodic solutions to the model, as well as new, quasi-periodic solutions. We classify and fully characterize the bifurcations yielding different states and analyse their stability. In doing so, we provide a clear picture of the full bifurcation diagram for the fundamental model of microtubule-motor protein complexes.

Relaxation dynamics of half-quantum vortices in a two-dimensional two-component Bose-Einstein condensate

Dr Hayder Salman

University of East Anglia

Abstract:

We study the relaxation dynamics of quantum turbulence in a two-component Bose-Einstein condensate containing half-quantum vortices. We find a temporal scaling regime for the number of vortices and the correlation lengths that at early times is strongly dependent on the relative strength of the inter-species interaction. At later times we find that the scaling becomes universal, independent of the inter-species interaction, and approaches that numerically observed in a scalar Bose-Einstein condensate.

The effect of contact line motion on the deposition of particles from an evaporating droplet

Dr Hannah D'Ambrosio

University of Strathclyde

Abstract:

The evaporation of sessile droplets occurs in numerous physical contexts, with applications in nature, industry, and biology. The evolution of, and deposition from, an evaporating droplet has therefore been subject to extensive investigation in recent years, with particular interest in droplet lifetimes and the ring-like deposit ("coffee-ring") that often forms at the contact line of a pinned evaporating droplet. Previous work has shown that the mode in which a droplet evaporates is a key factor in determining the lifetime of a droplet undergoing diffusion-limited evaporation. However, few studies have investigated the effect of the mode of evaporation on the deposition of particles from an evaporating droplet, and those that do often use a spatially-uniform evaporative flux to approximate the diffusion-limited model. In this talk we investigate the effect of contact line motion and the local evaporative flux on the deposition of particles from an evaporating droplet. For a thin axisymmetric droplet, we determine the resulting flow due to evaporation, the evolution of the concentration of particles within the droplet, and the evolution of the mass of deposit on the substrate for a droplet undergoing diffusion-limited and spatially-uniform evaporation in four different modes of evaporation. We find qualitatively different deposit types depending upon the mode in which the droplet is evaporating, as well as on the local evaporative flux. In particular, we show that spatially-uniform evaporation is not an accurate approximation to the diffusion-limited model for the flow within, and deposition from, an evaporating droplet when the contact line is receding.

An analytical approach to the design of acoustic metamaterials and metasurfaces

Professor David Abrahams

Univeristy of Cambridge

Abstract:

In this talk I will outline a framework for describing the effective propagation or scattering response of two-dimensional composite bodies or surfaces composed of subharmonic resonator inclusions. We shall focus on acoustic waves, and employ as a Helmholtz resonator element, a rigid cylinder with a narrow notch (or neck) connecting to an interior cavity.

Multipole methods have been used to good effect to obtain the effective behaviour of cylindrical inclusions in the many areas of photonics and phononics. We examine this design element, the resonant cylinder, to create metamaterials and metasurfaces, via the combined methods of matched asymptotic expansions and multipoles.

I shall commence the talk by briefly summarising the low-frequency propagation characteristics of a two-dimensional array of circular cylindrical resonators, and extend this to the reflection and transmission properties of a surface containing such elements. I shall conclude by offering brief results and indicating the benefits of this approach to practical metamaterial design.

References:

Michael J. A. Smith and I. David Abrahams, Tailored acoustic metamaterials. Parts I and II, Proceedings of the Royal Society A478, 20220124 and 20220125, June 2022.

Michael J. A. Smith, Philip A. Cotterill, David Nigro, William J. Parnell and I. David Abrahams, Asymptotics of the meta-atom, Philosophical Transactions of the Royal Society 380, 20210383, October 2022.

Exploring the lived experiences of women mathematics PhD students

Dr Laura Wadkin

Newcastle University

Abstract:

Women remain under-represented in the mathematical sciences across UK Higher Education Institutions (HEIs). In our study we use qualitative methods to understand the thoughts, feelings and lived experiences of female mathematics PhD students. Through one-to-one interviews, we seek to understand how the participants feel their gender may or may not have impacted their experiences as PhD students, including their relationships with supervisors, their interactions with role models, their identity as a mathematician, and their post-PhD career choices. We identify six common themes significant to their PhD experiences in relation to their gender: a sense of not belonging, feeling treated differently by supervisors, a lack of exposure to female role models, experiences of microaggressions and sexism, a complex picture of professional self-identity, and gender as a barrier to an academic career. We explore participants' recommendations for improving research culture, which include increasing the number of female role models and mentors, and increasing the frequency of structured Equality, Diversity and Inclusion events. This work deepens our understanding of the lived experiences of female PhD students in male-dominated fields and will aid the development of initiatives to combat the gender-imbalance within mathematical sciences and to improve women's experiences.

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Stretching and breaking of polymeric nanofibre bundles

Professor Astrid de Wijn

Norwegian University of Science and Technology

Abstract:

Bundles of polymeric materials are ubiquitous and play essential roles in biological systems, and often display remarkable properties. With the never-ending experimental advances in control and manipulation of molecular properties on the nanometic level follows an increasing demand for a theoretical description that is valid at this scale. This regime of nano-scale bundles of small numbers of molecules has not been investigated much theoretically; here chain-chain interactions, surface effects, entropy, nonlinearities, and thermal fluctuations all play important roles.

I will present an exploration by molecular-dynamics simulations of single chains and bundles under different types of external loading. Stretching, breaking, and rearrangments of chains are investigated, as well as their nano-scale thermodynamics.

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Active matter dispersion with absorbing boundaries: Fourier methods to the rescue

Dr Hakan Caldag

York University

Abstract:

Shear induced dispersion of active matter is qualitatively distinct from the well-studied Taylor dispersion of passive particles. The distinctions mainly stem from biased particle motility and complex boundary interactions. Aris's method of moments has been hugely successful in predicting the effective drift and diffusion along channels under no-flux conditions, but struggles when the flux is non-zero. Non-zero flux has practical relevance in terms of biofilm formation in bioreactors, which leads to a reduction in their efficiency. Tracked particles are not conserved in such systems, resulting in cumbersome calculations at higher orders. Here, we develop a Fourier approach that can side-step some of the complexities in Aris' method to tackle a non-trivial class of active media problems with taxes and boundary interactions. The method is efficient and accessible. We challenge the predictions asymptotically with known results for leaky pipes with cross-flows and numerically with Lagrangian simulations, providing very good agreement for long-time and transient solutions. We find that a strict ordering of eigenvalues is unnecessary to derive meaningful analytical results. Interestingly, for the case of two absorbing walls with biased motion towards one, analysis reveals an optimal taxis strength for axial dispersion that desensitizes dependence on absorption rates. The Fourier approach opens a path for improved qualitative interpretation of results for this wide class of systems.

Adventures in Maths Communication

Dr Christian Yates

Bath University

Abstract:

Over the last few years I have been fortunate enough to do a considerable amount of mathematics engagement across a range of different media and with a range of different audiences. I've written articles for national (Guardian, Independent, New statesman, BBC Futures, The Huffington Post, IFL science and Byline Times) and international (Scientific American, China News, Ouest-France) news outlets as well as providing expert commentary to all major UK newspapers. I've appeared on TV and Radio both at home (Newsnight, Panorama, BBC's More or Less, Inside Science, BBC News at 6) and abroad (CNN, Euronews, DW). I've done podcasts (Guardian Science podcast) and youtube videos (numberphile). I've sent mathematics viral on social media and I've written two popular maths books. As a member of Independent SAGE I have presented mathematics live to audiences in the hundreds of thousands and I've given scientific advice directly to policy makers in parliament.

In this talk I'll give an overview of some of my communication activities, sharing the highs and the lows. I'll explain the paths by which I came to these many different activities and try to distil the lessons that I've learned, both positive and negative, so others might learn from my experiences (and avoid my mistakes).

Cell-level modelling of active forces in early-stage development

Professor Rastko Sknepnek

Dundee University

Abstract:

Gastrulation is an essential, highly conserved process in the development of all vertebrate embryos, including humans. It involves large-scale cell and tissue movements. When not executed properly, it can lead to a wide range of congenital defects, or, in more extreme cases, cause abortion of development. Gastrulation requires the integration of critical cell behaviours such as cell differentiation, division, and movement through chemical and mechanical cell-cell signalling, to achieve the morphogenesis essential for proper functions. These interactions between signalling and cell behaviours create complex feedback loops between tissue, cell, and molecular length- and timescales that have evolved to enable robust formation of complex multi-cellular structures. In this talk, using the vertex model for cell-level description of epithelial tissues, we will discuss how various forms of active processes, such as mechano-chemical feedback, cell growth, division, ingression, etc. couple to cell mechanics and lead to patten formation and flows in model tissues. We will also make qualitative comparisons to the primitive steak formation (i.e. the gastrulation) in chick embryos.

How to be an Ally: a collaborative definition of allyship in STEM

Miss Rosie Evans

University of Birmingham

Abstract:

"We define male allies as members of an advantaged group committed to building relationships with women, expressing as little sexism in their own behavior as possible, understanding the social privilege conferred by their gender, and demonstrating active efforts to address gender inequities at work and in society." - Harvard Business Review

Do you agree/disagree with this statement? Is there anything missing or that you would add?

In October 2022, I co-hosted a workshop for postgraduate research students within the College of Engineering and Physical Sciences at the University of Birmingham entitled "How to be an Ally". The workshop aimed to collaboratively create a definition for what we as postgraduate research students believe to be an ally, with a focus on gender equity. We also planned to generate a broad set of actions to promote and develop allyship across the college ranging from individual to institutional level change.

In this talk, I will introduce the concept of allyship, and what it means to be an ally in the context of academia and more specifically STEM subjects. I will talk about the objectives of the "How to be an ally" workshop and discuss some of the allyship actions suggested by attendees. Finally, I will discuss why we think the set-up of this workshop worked well and how this could be taken further in the future to involve more staff participation and other institutions.

Elastic Bistability and the Geometry of Cellular Neighbourhoods in Choanoflagellates and Green Algae

Professor Raymond Goldstein

University of Cambridge

Abstract:

This talk will describe two recent advances in understanding the physics of cellular organization in simple multicellular organisms. The first part concerns the fluid dynamical and elastic properties of the recently discovered multicellular choanoflagellate C. flexa, which dynamically interconverts between two hemispherical forms of opposite curvature. The swimming and filter-feeding properties are described within a simple model of a raft of spheres with associated stokeslets to represent the action of the flagella. An elastic model based on linear elasticity of the microvilli of adjacent cells that adhere to each other is shown to support bistability at the organism level as a consequence of the presence of numerous pentagonal neighbourhoods in the raft. In the second part I will first review the recent findings that the cellular neighbourhood volumes in both lab-evolved and extant multicellular species, obtained by Voronoi tessellations based on the cell locations, are accurately described by gamma distributions, suggesting a hitherto unrecognized "universal" aspect of noise in cellular packing. Here we propose an explanation of those observations by considering the very simplest models for stochastic ECM generation by somatic cells and show that they define Poisson point processes whose Voronoi tesselations are demonstrably governed by gamma distributions. I summarize by proposing a link between the two parts of the talk.

Computing Lagrangian Coherent Structures without particle tracking

Dr Hossein Amini Kafiabad

Durham University

Abstract:

The invariants of the Cauchy–Green strain tensor are frequently use to detect Lagrangian Coherent Structures, which are material geometries in the fluid flows with optimal properties such as stretching/repelling. More particularly, the finite-time Lyapunov exponent (FTLE), which is an eigenvalue of the Cauchy–Green tensor, is extensively used for understanding the Lagrangian behaviour of a wide range of unsteady flows. Typical approaches for computing the Cauchy–Green tensor require tracking a large number of particles to construct the flow map gradient for a finite interval. This has drawbacks that include large memory demands (time series of velocity fields needs to be stored) and complications of parallelisation. We develop a novel approach in which the Cauchy–Green strain tensor is computed by solving partial differential equations (PDEs). These PDEs can be discretised in a variety of ways and solved simultaneously with the dynamical governing equations (e.g Navier Stokes) to minimise the memory footprint

An assessment of the contact rates between individuals when movement is modelled by a correlated random walk

Dr Joseph Bailey

Essex University

Abstract:

Understanding how individuals come into contact with each other is important in many fields from biology and ecology to robotics and physics. However, in many applications the underlying mode of movement is not considered and, instead, contacts are considered as a fraction of all possible contacts amongst a population. This gives rise to the mass-action law and a negative quadratic relationship between contacts and individuals. Here we consider how a simple movement model, the correlated random walk (CRW), affects the contact rate in a standard Susceptible-Infection (SI) model. We show that the contact rate is not always well described by the negative quadratic relationship, I(N-I) (where I is the number of infected at a given time and N the total number of individuals). Instead, we find that a contact rate proportional to $[I(N-I)]^{\alpha}$ with $0<\alpha\leq 1$ is a better qualitative fit, where α depends upon parameters such as the straightness of the movement and the density of individuals.

In this talk we present these findings, as well as discuss the potential importance of including specific models of movement in other natural ecological phenomena, which usually consider only very simplified movement models.

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Spatial patterns and multi-stability in non-local models of interacting species

Dr Valeria Giunta

Swansea University

Abstract:

Understanding the mechanisms underlying the spatial distribution of organisms is a central issue in both ecology and cell biology, and mathematical models can help to provide a deeper insight.

In nature, every individual, be it a cell or an animal, inspects its territory before moving. The process of acquiring information from the environment is typically non-local, i.e. individuals have the ability to inspect a portion of their territory.

In recent years, a growing body of empirical research has shown that non-locality is a key aspect of movement processes, while mathematical models incorporating non-local interactions have received increasing attention for their ability to accurately describe how interactions between individuals and their environment can affect their movement, reproduction rate, and well-being.

In this talk, I will present a study of a class of advection-diffusion equations that model population movements generated by non-local inter- and intra-species interactions. Using a combination of analytical tools, I will show that these models support a wide variety of spatio-temporal patterns that are able to reproduce segregation, aggregation and time-periodic behaviours commonly observed in real systems. I will also show the existence of parameter regions where multiple stable solutions coexist and hysteresis phenomena. Finally, I will analyse the phase transitions driven by non-local interactions. All the analytical results are supported by numerical simulations.

Overall, I will describe various methods for determining the pattern formation properties of these models, which are fundamental for answering the question of how organisms position themselves in space.

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Instability of stratified and diffusive Taylor-Couette flow

Dr Junho Park

Coventry University

Abstract:

In fluid flow with heat transfer, thermal diffusion is characterised by the Prandtl number Pr, which is the ratio between fluid kinematic viscosity and thermal diffusivity. Understanding the role of thermal diffusion is important for engineering and naturally occurring systems where the Prandtl number varies as Pr ~ O(1) for the air, Pr ~ O(10^(-2)) for liquid metals or the liquid metal core of the Earth, or Pr ~ O(10^(-6)) in the interior of the Sun and stars. In stably stratified flows, it has been known that high thermal diffusivity at low Pr suppresses the stratification effect. Our study aims to explore the effect of thermal diffusion in the context of Taylor-Couette (TC) flow with axial stratification. We first conduct linear stability analysis with a particular focus on Prandtl-number dependence and demonstrate how centrifugal instability of stratified TC flow is affected by the high thermal diffusivity. We will also describe a self-similar behavior observed for this TC flow in the limit of strong stratification and thermal diffusion. Furthermore, there will be discussion on nonlinear phenomena such as saturation, secondary instability or laminar-turbulent transition under the effect of strong thermal diffusion.

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Topological modes in stellar oscillations and instabilities

Mr Armand Leclerc

University of Lyon

Abstract:

Topological physics was created in condenser matter, in order to explain certain anomalous conductive modes in insulators. It has been shown that this framework applies in fluid wave problems, and explains the origin of fundamental modes crossing frequency gaps in inhomogeneous media. Using microlocal analysis, one can show that certain peculiar modes of stellar oscillations or instabilities can be explained by topological analysis.

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Monopoles, Alice rings and non-Abelian vortices: topology and dynamics in spinor Bose-Einstein condensates

Dr Magnus Borgh

East Anglia University

Abstract:

Atomic spinor Bose-Einstein condensates have become prime testbed systems for the exploration of topologically complex defects and textures resembling those found in superfluid liquid He-3 and quantum field theories. They exist in a rich variety of magnetic phases with different order parameter symmetry support a wide range of different topological defects and textures, from quantised vortices to Skyrmions. Here I will show how spinor Bose-Einstein condensates are promising candidates for realisation of non-Abelian vortex dynamics. In media with point-group order-parameter symmetries, the topological charges of line defects may not commute. As a result, colliding vortices cannot reconnect without forming a connecting rung vortex. Such vortices have analogues from cosmology to liquid crystals. I will further demonstrate how analogues of magnetic monopoles exhibit a rich topological dynamics that include oscillations between spin-Alice-ring and split-core configurations.

References:

[1] M. O.Borgh and J. Ruostekoski, "Core Structure and NonAbelian Reconnection of Defects in a Biaxial Nematic Spin2 Bose-Einstein Condensate", Phys. Rev. Lett. 117, 275302 (2016).

[2] Y. Xiao, M. O. Borgh, A. Blinova, T. Ollikainen, J. Ruostekoski, and D. S. Hall, "Topological superfluid defects with discrete point group symmetries", Nat Commun. 13, 4635 (2022).

[3] G. Baio and M. O. Borgh, "Composite cores of monopoles and Alice rings in spin-2 Bose-Einstein condensates", arXiv:2401.04103.

Integrating wastewater data and public health data for cost-effective COVID-19 surveillance

Dr Guangquan Li

Northumbria University

Abstract:

During the COVID-19 pandemic, studies in several countries have demonstrated wastewater-based epidemiology as an efficient surveillance tool to monitor the disease. To date, much of the existing work has focused on modelling the SARS-CoV-2 RNA concentration in wastewater at the level of sewage treatment work, where wastewater samples were obtained and concentration of viral contents was measured. Only a few predict wastewater viral concentration at a fine spatiotemporal resolution over an entire country, a necessary step to use wastewater data for early detection of local outbreaks. In this talk, I will describe how we predict weekly wastewater viral concentration for the whole of England at fine spatial scales using a geostatistical model. I will also discuss a data integration approach that combines the space-time wastewater predictions with the prevalence estimates obtained from public health data collected through randomised surveys and diagnostic testing. Through this framework, I will demonstrate the important role that wastewater data can play in the post-pandemic era where collection of public health data on COVID operates at a reduced capacity.

A computational approach to estimate the effect of crop diversification in future climate scenarios.

Dr Marianna Cerasuolo

Sussex University

Abstract:

Climate change will likely influence various aspects of agricultural production, encompassing temperature and precipitation fluctuations that could impact nutrient cycling, soil moisture, pest occurrences, and plant diseases. Crop diversification and multiple management practices are being advocated as viable solutions to counteract the adverse impacts of climate change on agricultural systems. The amount of carbon in soil is often used as a proxy for soil health, and increasing the capture and storage of carbon dioxide has the potential to make a positive impact on mitigating climate change. However, the soil carbon dynamics is slow and data-informed mathematical models are necessary to support research in agriculture, for example, to predict the impact of innovative cropping strategies.

In this talk, I will present the experiment-based mathematical model ECOSSE-M developed within the Diverfarming project. Focussing on the region of the Po Valley in Northern Italy, I will show how, through the use of statistics and machine learning algorithms, we were able to analyse the spatial dynamics of the soil carbon in intensive cropping systems, measure the effect of different managements, and identify those factors that drive the increase in stored carbon. This study represents the first in which science-based information on the sustainability of crop diversification in future climate change scenarios is analysed.

The impact and cost-effectiveness of pneumococcal immunisation strategies for the elderly in England

Dr Jasmina Panovska-Griffiths

University of Oxford

Abstract:

We compare the impact and cost-effectiveness (CE) of vaccination against pneumococcal disease with the existing pneumococcal polysaccharide vaccine (PPV23) to the new 15-and 20-valent pneumococcal conjugate vaccines (PCV15 and PCV20), targeting adults aged 65 or 75 years old.

A static Markov model for immunisation against pneumococcal disease, capturing different vaccine effectiveness and immunity waning assumptions, projected the number of IPD/CAP cases averted over the thirty years following vaccination. We combined it with an economic model and probabilistic sensitivity analysis to evaluate the CE of the different immunisation strategies at current vaccine list prices and the willingness-to-pay at a median threshold of £20,000/QALY and an uncertainty threshold of 90% of simulations below £30,000/QALY.

PCV20 averted more IPD and CAP cases than PCV15 or PPV23 over the thirty years following vaccination at both ages under base vaccine effectiveness assumptions. At the listed prices of PCV20 and PPV23 vaccines as of May 2023, both vaccines were cost-effective when vaccinating 65- or 75-year-olds with an ICER threshold of £20,000 per QALY. To achieve the same cost-effectiveness as PPV23, the additional cost of PCV20 should be less than £44(£91) at an ICER threshold of £20,000/QALY (£30,000/QALY) if vaccination age is 65 (or £54(£103) if vaccination age is increased to 75).

We showed that both PPV23 and PCV20 were likely to be cost-effective. PCV20 was likely to avert more cases of pneumococcal disease in elderly adults in England than the current PPV23 vaccine, given input assumptions of a higher vaccine effectiveness and slower waning for PCV20.

How wetting affects the evaporation of droplet arrays

Dr Alex Askounis

University of Wolverhampton

Abstract:

Droplet evaporation is ubiquitous in everyday life with considerable research focusing on how aspects such as substrate thermal conductivity, roughness, and wetting, vapour content and more. Nonetheless, limited work focused on how droplet arrays evaporate and the mechanics remain little understood. Some experimental works have shown the potential for applications in biomedicine and sequencing. A first framework on droplet array evaporation was proposed by Wray et al., but focused on the single case of hydrophilic substrates. In this work, we expand on that work and consider different wetting scenarios on different array configurations. We solved Fick's laws using 3D steady-state numerical model to derive the contact angle function. Computations were validated against experiments, showing an agreement with previous hydrophilic model. On hydrophobic substrates, the contact angle is much lower than unity which indicates low evaporation rates for our arrays.

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Inclusive STEM Teaching in the Age of the Open Web

Professor Volker Sorge

University of Birmingham

Abstract:

Making content in STEM subjects (Science, Technology, Engineering and Mathematics) accessible is a very challenging task. It is not only labour intensive and often reliant on expensive, niche products, but the results are sometimes sub-optimal and not easily transferable between learners and their different needs as or reliant on particular software platforms and hardware devices. However, with more an more teaching material being made available on the Web, their is a great opportunity to exploit the Web's rich environment of open standards and its culture of open source development to enable a fully inclusive learning and teaching experience.

I shall present a number of solutions for making STEM content accessible automatically for readers with visual impairments and learning disabilities. The key is sufficiently rich semantic interpretation that is produced algorithmically and embedded into the content's representation invisibly. In particular, I shall talk about the versatile rendering of mathematical formulas and the generation of accessible data visualisation straight from statistical models and demonstrate how they can be transformed into text, speech, sound and various tactile formats. I shall also discuss some of the obstacles that still hinder a truly inclusive education in Mathematics and present some ideas how we can overcome them.

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Parametrising dislocation dynamics from atomistic simulations with uncertainty

Dr Thomas Hudson

Warwick University

Abstract:

Modelling dislocation dynamics is crucial for the nanoscale prediction of plastic mechanisms in metals. In particular, the presence of phase boundaries in complex alloys can impede dislocation motion, and accurately describing and parametrising these effects would allow for the prediction of macroscale plastic properties. In this work, dislocation motion in pure Nickel was simulated using Molecular Dynamics (MD) simulations, and a simple ODE model for the dislocation motion was parametrised. To identify the parameters, a Differential Evolution Monte Carlo (DE-MC) approach was applied within a Bayesian framework to fit the parameters of an equation of motion to the dislocation trajectories extracted from the MD simulations. The parameters of interest are the effective mass, drag coefficient, and force experienced by the dislocation as the simulation cell is sheared and the dislocation moves. Parameterising dislocation motion in this way means that the fitting procedure yields physically meaningful parameters, which can then be used as inputs to larger lengthscale simulations. Moreover, using DE-MC sampling makes it possible to obtain parameter distributions and propagate uncertaintines through the model, quantifying the uncertainty in its predictions: the dislocation positions and velocities. This work serves as a first step towards building a more comprehensive surrogate model that can describe the deformation behaviour of superalloys.

Abstracts of Posters

Phase-Isostable Reduction of Oscillatory Neural Mass Networks with Delays in local dynamics and network connections

Mr Robert Allen

University of Nottingham

Abstract:

Large-scale brain signals that are seen in EEG and MEG recordings are believed to derive from the combination of local synaptic activity and non-local delayed axonal interactions. Considering networks of neural mass models, two sources of delays may be present; within node delays which can induce oscillations, and delayed network interactions between nodes which can strongly influence phase-locked network states and their bifurcations.

To understand how delays influence patterns of phase-locked states we aim to derive network equations for the phase and isostable coordinates of oscillations of each node, an extension of out previous work to include delays. This requires knowledge of the phase and isostable responses of the delay-induced oscillations of the nodes to perturbations in addition to incorporating coupling delays into the phase-isostable interactions.

We show how the response functions may be computed using a harmonic balance method, providing an improvement in efficiency and accuracy on standard solvers for delay differential equations

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Dam-Break Flows: down a hillside and surging up a beach

Dr Mark Cooker

University of East Anglia

Abstract:

When a dam breaks, the pent-up water accelerates from rest, due to gravity. The early-stage flow is modelled [1] as inviscid and irrotational, using a potential. The theory fills a gap in nonlinear shallow-water theory, NLSW, which struggles to describe early-stage motion. For a dam break starting at t=0, hydrostatic pressure immediately changes to a dynamic pressure, p. While t is relatively small, p obeys Laplace's equation subject to a linear bed condition, and p vanishes on the free surface. Fully two-dimensional flow evolves until the velocity becomes nearly parallel to the bed, after which NLSW can take over. The acceleration field, A, is greatest at the toe of the exposed water face: e.g. for a 45-degree hillside, A is up to TWICE that of a point-particle sliding downhill. The model also describes uphill flow on a sloping beach. A surging wave is shown to have large toe acceleration [1]. New work will describe axisymmetric converging flow in a crater — precursor of a vertical jet.

[1] M.J. Cooker 2023 Exact solutions for the initial stage of dam-break flow on a plane hillside or beach. J. Fluid Mech-Rapids, 972 R7, 11pp: https://doi.org/10.1017/jfm.2023.752

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Extensible-pendulum and double-pendulum problems: damping & periodic forcing, chaos & fractals

Dr James Christian

Salford University

Abstract:

The double pendulum is one of the most famous examples of a deterministic system that can exhibit wildly unpredictable––chaotic––motion. Slightly less well-known is the extensible pendulum, where a mass on a spring vibrates in the vertical plane [H. N. Núñez-Yépez et al., Phys. Lett. A, vol. 145(2,3), 101–105 (1990)]. Even though the spring is linear in nature (its restoring force is always proportional to extension), the extensible pendulum can become highly nonlinear when Hooke's law operates in two dimensions––Pythagoras's theorem imposes, in an essential way, a mathematical nonlinearity that has an easily-interpretable geometric character. Both systems may be modelled by deploying Lagrangian dynamics. In the best-known cases, they execute free oscillations prescribed by trajectories inside a four-dimensional phase space.

Here, we consider the extensible- and double-pendulum problems in the presence of two additional simultaneous effects: damping and external forcing. The former is introduced into the Euler-Lagrange equations by way of a velocity-dependent potential; the latter, by moving the suspension point in some prescribed fashion. Perhaps surprisingly, neither of these effects appears to be well-explored in the literature. Our interest lies with periodic excitation in the vertical or horizontal directions, or in a circular motion. When subject to external (i.e., explicitly time-dependent) forcing, the systems become non-autonomous and the solution complexity is increased accordingly. Here, we will detail the derivation of the equations of motion and present a range of new results from recent computations. Our study uncovers limit-cycle attractors and period-doublings, bifurcation cascades, and dissipative chaos.

This is joint work with Mr Mehrdad Jafari and Mr Dennis Horne

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Travelling fronts in a generalised neural field model that couples to the extracellular space

Mr Oliver Cattell

University of Nottingham

Abstract:

Continuum models of cortical activity often take the form of integro-differential equations known as neural fields. These models are known to support travelling fronts, with front speed strongly dependent on the threshold of the firing rate activation function. However, these models ignore ion exchange with the extracellular space, which is known to play a role in several pathological behaviours. For example, in diseases such as migraine or epilepsy, large slow-moving waves in the extracellular ion concentrations known as spreading depression occur. The large concentration gradient induced by such a wave can force neurons into depolarisation block and prevent normal firing.

To explore the role that the extracellular space has on the spatiotemporal patterns seen in cortex, we construct a simplified model of a scalar neural field with a Heaviside firing rate and threshold that is dependent upon ion concentration in the extracellular space. This is a proxy for changes in neuronal excitability driven by extracellular ionic concentrations. The transport of ions in the extracellular space is assumed to be via diffusion. We derive an expression for the front speed using an interface dynamics approach and show how the front speed is modulated by the rate of diffusion.

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Pattern formation driven by three-wave interactions with two critical wavenumbers

Miss Laura Pinkney

University of Leeds

Abstract:

Three-wave interactions (or resonant triads) arise from a set of three wavevectors where the sum of first two gives the third wavevector. They are present in a number of systems and can be used to explain pattern-forming behaviour in the Faraday wave experiment and in coupled Turing systems. As the lowest order nonlinear interaction, they often govern the dynamics close to onset. Considering problems with two critical wavenumbers, it is possible for two waves of a longer wavelength to interact with a third wave of shorter wavelength, and vice versa. Resonant triads with these two length scales can explain the presence of a wide variety of patterns such as stripes, hexagons and rhombs, in addition to more complex structures such as superlattice patterns and even spatio-temporal chaos when many resonant triads interact with one another. Indeed, rotations of a single triad (by the angle separating these wavevectors) generates a second resonant triad, which can interact with the first due to the shared wavevector.

In this poster we present an analysis of the patterns formed by a model PDE, an adaptation of the Lifshitz—Petrich equation with additional nonlinear terms, which itself is an extension of the Swift—Hohenberg equation with two critical wavelengths. Our PDE model allows for an infinite number of wave interactions unlike an ODE approach, which is limited by the size of the system.

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Optimal experimental design for quantification of uncertainty in models of hERG binding mechanism

Mr Frankie Patten-Elliott

University of Nottingham

Abstract:

The rapid delayed rectifier potassium current (IKr) carried by the human Ether-à-go-go-Related Gene (hERG) channel contributes to the regulation of the cardiac action potential duration in cardiac muscle. A variety of pharmaceutical compounds are responsible for a reduction in IKr and this can lead to an increased risk of cardiac arrhythmia. Quantifying uncertainty in the compound-specific hERG binding mechanisms can help to assess this drug-induced proarrhythmic risk. In this work, we investigate optimising experimental voltage protocol designs to differentiate between several proposed models of binding mechanisms. Data were recorded using the SyncroPatch 384 automated patch clamp instrument under drug-specific optimised protocol designs. By fitting proposed binding models to this collected data, we were able to use model selection and uncertainty quantification techniques to rule out some models as implausible and increase our understanding of the underlying mechanisms at play.

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Coarsening and pattern formation in solids

Miss Sakina Abdul Manan

University of Leeds

Abstract:

Coarsening is a process where small features agglomerate together to form larger and larger features, eventually limited only by the size of the system. It can happen in materials science, where small voids in a metal join together to create large voids, eventually causing brittleness. In contrast, in pattern formation, structures (stripes and hexagons) form with a characteristic length scale largely independent of the size of the system. We examine a two component Cahn-Hilliard model that shows a transition from coarsening behavior to pattern formation. We study the linear and nonlinear features of this model to explore this transition.

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Modelling bacterial chemotaxis and chemokinesis in dynamic environments with application to bacteria-root interactions

Mr Jason Bains

Newcastle University

Abstract:

Bacterial chemotaxis, the bias of swimming motions by chemical gradients, is well understood in purely spatially varying chemical profiles, and the chemotactic drift speed, the average velocity at which bacteria migrate up gradients of attractant or down gradients of repellent, has been found analytically. However, the motile response of bacteria to chemical profiles that change in both space and time is not well understood. Critically, an expression for how the drift velocity depends on both spatial and temporal gradients has not been derived. Here, we achieve this using Monte Carlo Simulations to simulate different bacterial paths and average over these paths to obtain the mean drift velocity. Chemotaxis and chemokinesis, a chemical-induced swimming speed change, are important in bacterial interactions with the region around roots, known as rhizosphere. Bacterial colonisation of roots in the rhizosphere is essential to plant health, either by symbiotic (beneficial) or pathogenic (deleterious) interactions. We have also modelled these interactions, particularly how bacterial populations distribute around roots for given root geometries, growth patterns and chemical exudations. Our model of root growth, chemical transport, and bacterial swimming motion, which improves on those in the literature in terms of biophysical realism, elucidates the conditions in which root colonisation will occur. We will discuss numerical simulations exploring a range of bacteria-root interaction scenarios.

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Modelling of long three-dimensional surface waves on currents

Mr Benjamin Martin

Loughborough University

Abstract:

Two different classes of waves can be found from the 2+1-dimensional cKdV-type equation depending upon whether the coefficients are evaluated using the general, or singular, solution of R.S. Johnson's "generalised Burn's condition." From the general solution, plane waves that propagate at an arbitrary angle to the parallel current are obtained. Conversely, with the singular solution (envelope of the general solution), ring waves propagating over such currents are obtained. We numerically study the stability of the line solitons to the three-dimensional transverse perturbations in this regime. We also model the propagation of hybrid waves consisting of an arc of a ring wave connected to the plane waves.

This is joint with Dmitri Tseluiko, Sergey Tkachenko and Karima Khusnutdinova

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Constrained consensus-based optimization via reflected stochastic differential equations

Mr Piers Hinds

University of Nottingham

Abstract:

We consider the global non-convex optimisation problem in a bounded domain. We develop numerical algorithms for the problem, based on an interacting particle system described by reflected stochastic differential equations. We study well-posedness of the particle system as well as of its mean-field limit. Several numerical examples are presented.

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Evolution of quasi-periodic internal waves with rotation

Mr Korsarun Nirunwiroj

Loughborough University

Abstract:

Korsarun Nirunwiroj, Dmitri Tseluiko, Karima Khusnutdinova

We consider internal waves in a two-layer fluid with rotation using the f-plane extension of the Miyata-Choi-Camassa (MCC) model. We construct weakly-nonlinear solutions leading to the Ostrovsky equation free from zero-mean contradiction, and accommodating shear inertial oscillations and barotropic transport. By using the constructed solutions, we investigate the effect of rotation on the evolution of several classes of rather general quasi-periodic initial conditions.

The combined effects of heterogeneous susceptibility, non-pharmaceutical interventions and viral evolution on epidemic trajectories

Mr Ibrahim Mohammed

Strathclyde University

Abstract:

It has recently been established that individual variation in susceptibility lowers herd immunity thresholds. Using an SEIR model with gamma distributed traits it was shown that variation in susceptibility flattens epidemic curves in a natural way – an effect that can sometimes be misattributed to interventions. Given epidemic data in which non-pharmaceutical interventions (NPIs) were imposed to control an outbreak, estimating both the effect of the intervention and the coefficient of variation of susceptibility becomes complicated as both perform a similar task of flattening the curve. Our research seeks to address this challenge by conducting simulated epidemic studies and statistical inference.

We simulated epidemics of a respiratory viral disease such as covid -19 using known values of parameters of interest from literature to represent the disease dynamics, heterogeneity, and the impact of NPIs. For each combination of parameters 250 data sets were simulated and we fitted models with heterogeneity in susceptibility to estimate the basic reproduction number, coefficient of variation, and intervention parameters while keeping other parameters constant. Our results indicate that there exists some degree of correlation between the coefficient of variation and the non-pharmaceutical intervention, but this can be overcome by creative study designs.

Measuring Microplastics: Advancing Size Quantification through Enhanced Scanning Electron Microscopy

Mr Imoleayomide Ajayi

Loughborough University

Abstract:

Our world is increasingly permeated by microplastics and nanoplastics found both in the environment and even in our bodies. These plastics, especially those at the nanoscale (<=100nm), are a growing concern due to their potential ecological and health impacts. A key factor in understanding these impacts is accurately measuring the size of these microplastics. However, size measurement at such small scales is complex and can be skewed by the techniques used in imaging, such as Scanning Electron Microscopy (SEM). In this talk, we will discuss how to refine SEM imaging to provide more accurate size measurements of microplastics. By developing a statistical model to better understand and capture secondary electron emissions and detection in SEM, we aim to enhance the resolution and accuracy of microplastic size quantification. This improvement is not just a technical advancement; it is crucial in shaping our understanding of microplastics' role in environmental health and aligning with global efforts towards a net-zero future. Accurately measuring microplastics helps us comprehend their behaviors and potential risks, contributing to informed strategies for managing and mitigating environmental pollution.

Winding and Magnetic Helicity in Periodic Domains

Mr Daining Xiao

Durham University

Abstract:

In Cartesian geometry, the topological complexity of a given magnetic field can be quantified by its magnetic helicity which is equivalent to the flux-weighted Gauss winding numbers of all magnetic field lines. In this work, we present a generalisation of this classical result in three-dimensional domains that are periodic in two lateral dimensions, thus solving the open problem posed by M. Berger (J. Geophys. Res., 102, 1996). Such a domain choice is often employed in numerical studies in magnetohydrodynamics and is topologically non-trivial as a 2-torus. We propose periodic analogues of Cartesian winding numbers of open curves and the poloidal-toroidal decomposition of magnetic fields. This allows the identification of a particular vector potential that recovers the winding interpretation of magnetic helicity in periodic domains. Key properties of the winding-based magnetic helicity are also proved, including its time-conservation in ideal magneto-hydrodynamical flows, its connections to existing Fourier approaches, and its relationships to gauge transformations.

Sobol sensitivity analysis of action potential model for rabbit ventricular myocyte

Mr Zhechao Yang

Glasgow University

Abstract:

In this paper we rank the parameters of a widely used model of the rabbit cardiac action potential in increasing order of infuence on the outup of the model. This is a critical step in the pipeline of further applications of this model, for example for model reduction, parameter ingerence and simulation etc. Model of cardiac action potential describes the change of transmembrane potential via the time, consisting of the process of depolarization and repolarization. This model has gained widespread popularity across various applications, including but not limited to predicting heart diseases and more.

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Modelling Tumour Escape Mechanisms in CAR T-Cell Treatment of Leukemias

Mr Alexis Farman

University College London

Abstract:

A novel form of cancer immunotherapy, known as CAR T-cell therapy, has sparked significant enthusiasm among researchers and oncologists due to its effectiveness in treating various blood cancers.

In leukaemia patients, the application of this therapy triggers a massive expansion of CAR T-cells, leading to a rapid reduction of cancer cells to undetectable levels. Subsequently, the CAR T-cell population contracts, entering a sustained low-level state, primarily composed of Central Memory cells.

Clinical observations have indicated that the loss of this persistence may result in patient relapse, prompting extensive research to uncover why persistence is so crucial.

We employ Ordinary Differential Equations (ODEs) and stochastic methods to model the interactions between blast cells and immune cells, and investigate the mechanisms of loss of persistence and subsequent escape of tumour cells.

Understanding these mechanisms is crucial as achieving prolonged persistence of CAR T-cells proves very challenging.

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Modelling structure borne sound and radiation of submerged structures at high frequencies

Mr Samuel Palama

University of Nottingham

Abstract:

Predicting the vibroacoustic behaviour of structures in contact with water and at high frequencies can be a complex task due to the fluid-structure interaction. Computation time increases with frequency and the complexity of the structure. Being able to predict the sound radiated by a naval vehicle in contact with water could help in improving passengers' comfort as well as the impact on the marine fauna.

We use a prediction method called Dynamic Energy Analysis (DEA), so far emloyed mainly to assess the vibrational energy levels in the structure itself. DEA is a phase-space based ray tracing method determining energy densities in terms of the ray densities in a structure. The method can be applied on FEM-meshes and thus needs no SEA type substructuring.

We expanded the DEA methodology here to take into account radiation into the fluid in contact with the structure. From structure borne DEA calculations, we can determine directional sound radiation patterns both inside and outside the structure. We will demonstrate the new technique using some simple model systems.

An exact solution for laminar flow in fully-filled sewer pipes with egg-shaped and horseshoe-shaped cross-sections

Professor A. Lopes, Ms J. Zago Dias Feijó and Mr V. Coutinho da Silva

University of Brazil

Abstract:
The Poiseuille flow of a Newtonian fluid through an infinitely long fully-filled pipe with a constant cross-section is governed by a Poisson equation subject to Dirichlet boundary conditions. Whilst exact solutions for this problem are known for various geometries, only a select few are suitable for sewer pipe applications. This limitation primarily arises from the prevalence of unconventional cross-sections, such as egg-shaped and horseshoe-shaped configurations – which, in conjunction with the circular cross-section, constitute the three standard sewer cross-sections. The study of these complex geometries in the literature has predominantly depended on experimental, empirical, or numerical methods. The present work introduces a novel theoretical approach to Poiseuille flow within these geometries by means of a homotopy of two well-known polynomial solutions due to Boussinesq: The Poiseuille flow in elliptical and equilateral triangular pipes. Utilising a closed-form solution, we compute the flow rate, the friction factor–Reynolds number product, and the maximum fluid velocity for these oval configurations. The results are subsequently compared to a numerical solution using the finite-elements method, confirming that our exact solution can serve as a benchmark for future works involving pipe flows in such geometries.

Bifurcations of attractors in singular fast-slow system

Mr Said Elahjel

Exeter University

Abstract:
This project is going to examine bifurcations in multiple timescale dynamicl systems for number of fast and number of slow timescale of that dynamical systems which have subsystems and evolve on vastly different time.

Will try to develop and improve our mathematical understanding of attractors in both fast and slow subsystems and their interaction each other.

Attempt to classify Bifurcation of attractors for these dynamical systems, including relaxation oscillations, in singular limit systems with multiple fast and slow variables using a distinguished parameter approach.

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Modelling Epithelial Ridge and Sweat Gland Formation

Miss Luci Mullen

Durham University

Abstract:

We present a model for the formation of fingerprint ridges and sweat glands in mammals (mouse and human). We consider a two equation reaction-diffusion system representing the genes producing fingerprint ridges. Numerical simulations show that spatial solutions to these equations replicate the structure of observed fingerprint patterns (after modulation by initial and boundary conditions, as well as domain curvature). We also show how changes to the initial conditions for this model lead to changes in the solution structures, giving rise to 'arch', 'loop', and 'whorl' phenotypes. Finally, we consider several two and four species systems for modelling the distribution of sweat glands which form subsequently to the ridges but are constrained by them. The model of epithelial ridge formation corroborates recent experimental evidence regarding the molecular origin of these structures, whereas the extended sweat gland model suggests several avenues for further gene expression studies.

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A nonorthogonal geometric formulation of sheared rotation

Dr Gert Botha

Northumbria University

Abstract:

Sheared rotation is described by introducing a new nonorthogonal coordinate system where the shear is embodied by the geometry of the reference frame. This has the advantage that the effect of the shear is encapsulated in the vectorial operators, avoiding the need for ad hoc expansions of velocity or magnetic fields. In this newly defined reference frame our mathematical analysis identifies a previously unreported force that is due to the coupling of the gradient in the angular velocity with the axial component of the angular momentum. This force acts radially and its direction is determined by the angular velocity profile. Depending on the size of the angular velocity gradient, this force can be of the same order as the centrifugal and Coriolis forces.

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Thermal boundary layer around a circular cylinder on the seabed forced by free-surface waves

Mr Henry Thomas

Plymouth University

Abstract:

Many structures, man-made or otherwise, lie on the ocean floor and disturb the fluid and its properties. A case is presented for a circular cylinder that is either partially submerged or just touching a flat seabed. This has many applications for coastal engineering such as submarine pipelines, underwater compressed gas energy storage and underwater datacentres. The fluid is forced by the propagating free surface waves which determines the inviscid velocity profile around the cylinder. A theoretical model of the temperature field in the boundary region of the cylinder is presented and explored. It is found that steady streaming is a major factor in the role of heat transfer and is responsible for the transport of temperature into the bulk fluid. Thus, the temperature does not diffuse into the fluid uniformly but rather it is directed along the seabed in a jet-like manner. A numerical solver was used for comparison and to present the results graphically, visually displaying the temperature field. A parametric study into the effect of varying protrusion levels and frequencies was performed for applications in real-world scenarios. It was found that the heat flux decreases with higher frequency oscillations but reaches a maximum for an intermediate level of protrusion.

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Mathematical Modelling of Airways in Asthma Poster

Ewan Fewell

University of Nottingham

Abstract:

Inflammation, airway hyperresponsiveness and airway remodelling are key characteristics of asthma, but it is unclear how they are interconnected. Healthy airways exhibit a homoeostasis between cell proliferation, apoptosis, and extracellular matrix deposition such that there is no airway remodelling (i.e. the airway wall thickness remains constant), but it is not known how disruptions to these processes lead to pathologies associated with asthma. This project employs a simplified multiphase modelling approach (based on Hill et al. (2018)) to understand the key processes that support airway homoeostasis. The airway wall is modelled as a mixture comprising of four constituents: airway smooth muscle cells in either contractile (c) or proliferative (p) phenotypes, extracellular matrix (e), and tissue hydration (w). The airway wall is modelled as an axisymmetric thick-walled cylinder of fixed length. A homoeostatic state is one where the boundaries of the airway remain fixed. Together with recent experimental data from an in vivo mouse model of asthma describing airway wall composition, we employ this model to understand how perturbations to this homoeostatic state could drive airways into an asthmatic state, and ultimately to understand which processes are the cause of the disease.

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Single electrons on the Fibonacci quasicrystal: an interpolation between models

Professor Alastair Rucklidge

Leeds University

Abstract:

Quasicrystals are some of nature's most intriguing, and quite surprising, forms of matter. They are not periodic in space, nevertheless their constituent atoms and molecules are well-ordered in space. Here, we discuss the theory for the eigenstates and spectral properties of the the equation for single electrons in a quasicrystal. Bloch theory is the theory that governs eigenstates (the electronic wavefunction in quantum mechanics) in spatially periodic crystals, and is a cornerstone theory for understanding how electrons behave in periodic crystals. There is as yet no comparable theory for electrons in aperiodic crystals, and this is an area where mathematical and physical insight need to be combined. Three kinds of eigenstates are known: eigenstates for electrons that extend across the whole crystal, eigenstates that are exponentially localised in space, representing electrons that are caught by the aperiodic nature of the quasicrystal, and a third intermediate state with algebraic decay in space. We investigate this behaviour in a one-dimensional aperiodic crystal based on the so-called Fibonacci tiling. We use a new model that interpolates between representing atoms as a smoothly varying potential function and representing atoms as delta functions. Real atoms in real crystals are best described by something in between these limits, and our interpolating model allows two (and possibly all three) kinds of eigenstates, helping us understand how they are related.

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Solute transport in the cranial subarachnoid space

Ms Alannah Neff

University of Edinburgh

Abstract:

Cerebrospinal fluid (CSF) is a clear Newtonian fluid that surrounds the brain and the spinal cord. Specifically, it resides in the subarachnoid space (SAS), the area between the brain and a protective membrane called the dura. The CSF pulsates during the cardiac cycle, due to the changes of brain volume. This fluid plays a vital role in transporting essential nutrients and serving as a drainage pathway for harmful substances, including toxins and toxic proteins. Moreover, CSF can be used for drug delivery to the brain through a method called intrathecal delivery, where drugs are injected directly into the spine. The aim of this work is to understand the solute transport in the CSF for the applications in clearance of metabolic waste and drug delivery.

In this study we develop a simplified model of CSF flow and solute transport. We adapt a two-dimensional geometry that consist of two flat plates, representing the brain surface and the dura membrane. The bottom plate oscillates with a prescribed velocity. The depth of the subarachnoid space is much thinner than its length, which allows the use of lubrication theory. The solute transport is modelled with an advection-diffusion equation and we use an asymptotic expansion with respect to the aspect ratio of the domain to simplify it.

We obtain an analytical solution for the fluid flow at the leading order (oscillatory flow) and at the steady component of the first order flow (steady streaming). We then use these profiles to derive a simplified problem for the solute transport. This approach allows us to understand the fundamental mechanisms of solute transport in the CSF and provides a framework for investigating more realistic domain geometries and displacements.

Co-dimension 2 Bifurcation Analysis Around the Boundary Equilibrium Bifurcation in Impacting Hybrid Systems

Mr Hong Tang

Bristol University

Abstract:

BEB (Boundary Equilibrium Bifurcation) is one of the basic but interesting bifurcations in the non-smooth systems, where the equilibrium transition happens when interacting with those discontinuity manifolds. Apart from the static bifurcation, sometimes BEB also gives birth to limit cycle oscillation (LCO), which has been well studied in planar system and is termed as Hopf-like bifurcation by some researchers.

Recently, we formulated an implicit condition for the existence and stability of the LCO around the BEB in general n-dimensional impacting hybrid systems.

As a follow-up research, this paper tries to reveal some interesting co-dimension 2 bifurcation cases when the BEB happens along with bifurcation of the born LCOs. We start with the fold bifurcation, and we expect to obtain the same result with period-doubling and Neimark-Sacker bifurcations in place of the saddle-node bifurcations.

Modelling carbohydrate and protein metabolism in mammals

Mr Bandar Alharbi

Nottingham University

Abstract:

Enzymes that catalyse metabolic pathway reactions are regulated by hormones/growth promoters, gene overexpression/ knockdown or genetic mutations, all of which can affect essential cell functions such as growth. Our aim is to understand which parts of a metabolic network can be modified to influence growth, particularly of skeletal muscle. We propose a mathematical model based on coupled ODEs for the relevant metabolites and investigate the regulation of the system by considering the inputs and outputs from the network. We focus on cytosolic glycolysis, the mitochondrial TCA cycle, and the associated serine synthesis pathway. Our objective is to explore the consequences of overexpressing/ increasing the expression of two key enzymes, phosphoenolpyruvate carboxykinase 2, (PCK2) and phosphoglycerate dehydrogenase (PHGDH), on the TCA cycle and on serine production. We find a range of steady-states which depend upon input fluxes into the network. As input fluxes are altered, steady-states cease to exist due to the bifurcation to one of two states in which some metabolites grow linearly in time whilst others decay to zero. Leveraging asymptotic analysis, we approximate the asymptotic form steady-state solutions and so are able to find critical parameter values which determine where in parameter space the system's behaviour changes. Additionally, a parameter sensitivity analysis explores the effect of varying rate constants and input rates. Our numerical simulation results show that the up-regulation of PHGDH, the initial rate limiting enzyme in the serine-synthesis pathway, causes an increase in serine production.

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Optimal activation functions from an OPTA perspective

Dr Hui-An Shen

University of Bern

Abstract:

How static and dynamic stimuli are efficiently represented in the sensory neural system is a widely studied problem in computational and theoretical neuroscience. Here the neural system's task is to optimally estimate the said stimuli with noisy observations. In particular, due to the pulse-like sequences neurons generate, these noisy observations can be characterized by point processes, whose intensity is a function of the stimuli given by a non-linear map referred to as the "tuning function" or the "activation function". In this context, we can describe the estimation task as optimal Bayes-estimation of a static variable or optimal filtering of a dynamic variable under point-process observations. The diverse non-linear maps present in the sensory neural system were shown to be matched to the stimulus statistics, and was recently proposed as a functional classification for retinal ganglion cells in the visual system.

In this work, we develop a principled approach to derive a general expression for an "optimal" activation function subject to the underlying stimuli. We model the neuronal system as source coding with causal side information at the decoder, where the stimulus is viewed as the source and the side information consists of noisy observations, and define the rate-distortion problem in this setting. By introducing a definition for optimality based on OPTA (Optimum Performance Theoretically Attainable) in information theory, we obtain the general expression of the said optimal activation function taking the form of relative entropy between two probability distributions that arise from the solution to the said rate-distortion problem.

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Analysing Information Flow in Multi-View Neural Networks for Medical Imaging

Mr Lucas Farndale

University of Glasgow

Abstract:

Multi-view self-supervised representation learning is a vital method in medical image analysis, as it forgoes the large amounts of expensive and arduous manual labelling required by typical supervised approaches. These models are highly effective but generally suffer from a bias towards simpler features in data. One exciting avenue to improve such models is the distillation of privileged information (information available during training but not inference, e.g. more advanced imaging or sequencing methods) which enables models to be trained to find features which are more easily identified in the privileged information, without requiring ongoing production of the privileged modality for inference. This has exciting applications for clinical and research use, as it embeds information from state-of-the-art modalities in models for more routine techniques, however, existing approaches often reduce the models' ability to identify the simpler features which would be found in the absence of privileged information.

We show using an information-theoretic analysis that this is due to the multi-view assumption which is made implicitly or explicitly for all existing methods: that almost all meaningful semantic information is shared between views. We find that existing methods minimise useful predictive information as a consequence of this assumption and use this to propose a method which is robust to the multi-view assumption and achieves improvements of up to 101% on standard medical imaging tasks.

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MHD instabilities in stellar radiative regions

Virgin Durepaire

Observatoire de Paris - LERMA

Abstract:

Space-borne missions such as Kepler provided us with new observational constraints on stellar evolution. For instance, the slow-down of the core rotation of stars along with their evolution remains unexplained. As magnetohydrodynamic instabilities are potentially able to play a key role in the angular momentum transport inside stars, a comprehensive view of these instabilities is definitively needed.

In this context, our study focuses on - the magnetorotational instability (MRI) that is induced by shear within an axial (SMRI), toroidal (AMRI) or helical (HMRI) magnetic field - and the Tayler Instability (TI), a kink-type instability induced by a strong toroidal field. Considering the stratification and the thermal dissipation through a general approach provides us with a more realistic stability criteria.

We consider an azimuthal flow immersed in a magnetic field and used a local stability analysis (WKB approximation). For a distribution of modes, the sign of the growth rate characterizes the stability of the flow. We displayed regions of non-stability for different ranges of parameters, typical of stellar interiors and of MHD numerical simulation flows.

We have recovered results on MRI for axisymmetric perturbations, exhibiting the competition of the stratification and the shear through the ratio of the thermal and magnetic dissipation. We have shown that for realistic stratification, it would require a very high magnitude of the toroidal field to lead to the development of instabilities. We then studied the non-axisymmetric perturbations, we identified the regions of MRI and TI and how they merge for certain range of parameters.

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Predator-prey models and the hares-eat-lynx paradox

Mr Eduard Campillo-Funollet

Lancaster University

Abstract:

The hares-eat-lynx paradox is a phenomenon observed in a well-established Canadian hares and lynx dataset. Rather than following the classical predator-prey dynamics as expected from these species, in some occasions the prey peak follows the predator peak. In this talk, we will discuss how the observed dynamics can still be modelled by means of a predator-prey model with time dependent parameters, and we will present a robust inference scheme to find the optimal model parameters.

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Approach and separation of bundles of quantized vorticity

Dr Andrew Baggaley

Newcastle University

Abstract:

In the quasi-classical regime of quantum turbulence, it has long been hypothesised that there exists coherent vortical structures, made up of bundles of quantised vortices. More recently, there has been significant experimental evidence that points to their presence. Here, we perform the first quantitative study of the reconnection of bundles of quantised vorticity and show that the approach and separation of bundles during a reconnection is consistent with the symmetric $\omega \sim t^{1/2}$ scaling which is consistent with studies of individual quantised vortex reconnection and classical vortex reconnections. We also examined the phenomena of `bridge' structures that form between the vortex bundles during the reconnection process and have also been observed during the reconnection of classical vortices. We study their persistence and suggest that their dissipation is driven by vortex-vortex interactions within the bridge itself.

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Three-dimensional ship-wave patterns: A new computational approach

Mr Jack Keeler

University of East Anglia

Abstract:

The distinctive vee-shaped wave pattern that is observed in the wake of a moving object on the surface of a body of water is an intrinsically three-dimensional phenomenon and is a considerable challenge to calculate. We present a novel numerical formulation, based on a variational method, that captures this wave pattern for flow over a bottom topographic forcing or with a surface pressure distribution on the surface. This method, as well as being computationally competitive to other numerical schemes in the literature, allows us to i) robustly examine the steady bifurcation structure as system parameters are varied and ii) perform unsteady calculations efficiently. This poster describes this method and discusses future possible directions of research.

Poisson Scheme Structure Preserving Numerics for the Shallow Water Equations

Mx James Arthur

Exeter University

Abstract:

Most numerical schemes of the 2D Shallow Water Equations (SWE) struggle for long time prediction due to numerical error in the conserved properties of these equations. Arakawa & Lamb [1] found a finite difference approximation of the SWEs that conserve the finite difference approximations of energy and potential enstrophy. Then Salmon [2] produced a Geometrical generalised derivation of the Arakawa & Lamb scheme, which then allows for further work. Dellar and Stewart [3] then produces a scheme that has partial rotation and adds bottom topography.

We borrow ideas from Poisson Geometry such as the Poisson Scheme and their Casimir properties from Classical Mechanics to create numerical schemes for long term prediction of sloshing problems and add in the future add full rotation, forcing and bottom topography. We have derived second and fourth order schemes in space and use an RK4 scheme to time step. We will show some explicit second and fourth order schemes based on the Salmon paper and then some numerical results.

[1] - Akio Arakawa and Vivian R Lamb. "A potential enstrophy and energy conserving scheme for the shallow water equations". In: Monthly Weather Review 109.1 (1981), pp. 18–36.

[2] - Rick Salmon. "Poisson-bracket approach to the construction of energy-and potentialenstrophy-conserving algorithms for the shallow-water equations". In: Journal of the atmospheric sciences 61.16 (2004), pp. 2016–2036.

[3] - Andrew L Stewart and Paul J Dellar. "An energy and potential enstrophy conserving numerical scheme for the multi-layer shallow water equations with complete Coriolis force". In: Journal of Computational Physics 313 (2016), pp. 99–120.

Cyclic loading of a non-linear heterogeneous poroelastic material

Miss Zoe Godard

University of Oxford

Abstract:

Although tendon is widely modelled as a poroelastic material, to our knowledge there is no existing model exploring non-linear effects, and in particular how these modify the response in a "damaged scenario". Tendons are a highly connective tissue linking bone to muscle, playing a very important role in the ability to withstand load (for example during running), and they undergo large tensile stresses throughout their lifetime. Whilst tendon injury accounts for around half of reported muscoskeletal injuries, the underlying mechanisms are still not very well understood, despite efforts from a range of disciplines. We are interested in characterising the behaviour of tendon-like tissues in response to an applied cyclic load, and how this response is affected by heterogeneities in the stiffness and permeability, as experiment reveals altered values for these parameters in the diseased tendon. We present a non-linear poroelastic model for tendon, using Darcy flow and an appropriate non-linear constitutive law for the solid skeleton, and apply a cyclic tensile stress at the boundary, where frequency and amplitude are variable. Damage is represented by a local decrease in stiffness and/or permeability, and we are interested in the resulting strain and flux values, which affect the mechanical environment sensed by tendon cells. We present preliminary results to show how damage affects these values, and compare these to results for the linearised model, highlighting the importance of non-linearity in the dynamics.

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Surface instability of a finitely deformed magnetoelastic half-space

Mr Davood Shahsavari

University of Glasgow

Abstract:

We develop a mathematical model to analyse the surface instabilities of an incompressible magnetoelastic half-space. Based on a consistent variational formulation that accounts for magnetic fields inside and outside the material, we derive the instability criteria for developing surface wrinkles. We present new results for a Mooney-Rivlin-type constitutive model and show that the instability regime can be tuned by the applied magnetic field and constitutive parameters.

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Stabilization of cyclic processes by slowly varying forcing

Mr Julian Newman

University of Exeter

Abstract:

Constant external forcing of a dynamical system can sometimes induce stability via a saddle-node bifurcation involving the creation of a stable and unstable equilibrium. Similarly, it has been noted that slowly time-dependent external forcing of a dynamical system can induce stability. This latter stabilisation can be understood in terms the creation of a stable and unstable branch of the slow manifold, and we will consider how to define and analyse this bifurcation in analogy to the classical saddle-node bifurcation scenario. The created branch of the slow manifold has finite width over the slow-time axis, and accordingly we will see how this bifurcation manifests dynamically for finite-time dynamical systems on the circle. We will also compare the two types of bifurcation for a low-frequency-periodically forced system on the circle. This is joint work with Maxime Lucas and Aneta Stefanovska.

Lunch time talks

Digital learning workshop - assessment and accessibility

Christian Lawson-Perfect
Newcastle University

Abstract:

The award-winning Digital Learning unit in Newcastle University's School of Mathematics, Statistics and Physics develops and supports a range of open-source software for mathematics education. Numbas is a system for automated assessment of mathematical subjects and Chirun is a tool for producing course material in accessible formats, from LaTeX or Markdown source.

In this workshop we'll demonstrate Numbas and Chirun and invite attendees to discuss what they're doing, or would like to do, with online assessment and digital accessibility.

Organisations supporting Knowledge Exchange and Knowledge Transfer in the Mathematical Sciences: Are we meeting your needs?

Sophia Sanz Del Pino & Lauren Hyndman
Newton Gateway to Mathematics

Abstract:

In 2018 UK Research and Innovation (UKRI) published the independent review carried out by Professor Philip Bond, aimed at examining knowledge exchange in the mathematical sciences. Intended to "...highlight the importance of mathematical sciences in the UK society...", the report states, "Our vision is for the UK to become a world leader in generating economic and social benefit from MS. As the UK redefines its place in the world by capitalising on its strengths, the application and impact of world-class UK mathematical sciences has a key role to play. Knowledge is generated globally, and a vibrant KE culture engaging internationally will remain critically important to the UK."

The principal recommendation was that "a national centre in impactful mathematics for the UK should be created to work with industry and government to drive mathematical research through to commercialisation". This was achieved in 2022 with the creation and establishment of the KE Hub for Mathematical Sciences, initially funded by Isaac Newton Institute (Cambridge) that set aside EPSRC funds.

The knowledge exchange (KE) and knowledge transfer (KT) remits of these two organisations (the INI Newton Gateway to Mathematics) are complemented by the activity of both the International Centre for Mathematical Sciences (ICMS) and Innovate UK KTN. All four organisations work together to support the mathematical sciences community across the UK, but are we meeting the KE and KT needs and wants of the community?

Selecting and submitting to a scientific journal

The Royal Society

Abstract:

Hosted by the Royal Society, and aimed towards Early Career Researchers or those looking to publish their work for the first time, this lunchtime seminar will focus on how to successfully select and submit to a scientific journal. This talk will cover topics such as the factors to consider when choosing a journal, what happens once you submit your work, and how to navigate the submission process in order to achieve the best chance of success.